2020

PHYSICS

[HONOURS]

Paper : I

Full Marks : 75 Time : 4 Hours

The figures in the right-hand margin indicate marks.
Candidates are required to give their answers in their own words as far as practicable.

GROUP–A

1. Answer any **five** questions: 1×5=5

   i) Define moment of inertia tensor. Why it is a symmetric matrix?

   ii) Two quantities have the same dimensions. Do they represent the same physical content? Explain with examples.

   iii) What is Neumann's triangle?

   iv) Is the viscosity of a gas temperature dependent? If so, how does it vary?

   v) Define reciprocal vector and axial vector with example.

   vi) What is Newton's Collision rule?

[Turn over]

   vii) The position of a particle moving in a plane is given by $x = \sin\omega t$, $y = \cos\omega t$. Show that the trajectory repeats itself only if α is a rational number.

GROUP–B

2. Answer any **six** questions: 2×6=12

   i) Find the expression for the excess pressure inside a soap bubble over that outside by the method of dimensional analysis.

   ii) Deduce Laplace and Poisson's equations for gravitational potential and indicate in what cases they are applicable.

   iii) Show that the strain energy of a twisted wire is $\frac{1}{2}c_m\theta_m$, where c_m is the couple for the maximum twist θ_m.

   iv) Use Euler's equation and set up Bernoulli's theorem for steady streamline motion of an incompressible fluid.

   v) State and prove Kepler's second law of planetary motion.

   vi) Prove that

$$(\vec{A}\times\vec{B}).(\vec{B}\times\vec{C})\times(\vec{C}\times\vec{A})=\left[\vec{A}.(\vec{B}\times\vec{C})\right]^2.$$

vii) Two particles of masses m_1 and m_2 move in an arbitrary manner. If v_c is the velocity of the centre of mass and v the relative velocity of the two particles. Calculate the total kinetic energy of the system.

viii) Show that the angle of scattering in laboratory frame is maximum when the angle of scattering in centre of mass frame is $\cos^{-1}\left(-\frac{m_2}{m_1}\right)$, where m_1 and m_2 be the masses of colliding particles.

ix) State and prove perpendicular axes theorem for three dimensional body.

x) Write down the limitations of dimensional analysis of physical problems.

GROUP–C

Answer any **three** questions: 6×3=18

3. i) Find the eigenvalues and eigenvectors of matrix

$$A=\begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}.$$

ii) Define symmetric and anti-symmetric matrix with example. 1+(2+3)

4. i) Prove that $\operatorname{curl}\left(\frac{\vec{A}\times\vec{r}}{r^3}\right)=-\frac{\vec{A}}{r^3}+\frac{3\vec{r}}{r^5}(\vec{A}.\vec{r})$, where $\vec{A}$ is a constant vector and $\vec{r}=\hat{i}x+\hat{j}y+\hat{k}z$.

ii) If $\vec{A}=xz^2\hat{i}+2y\hat{j}-3xz\hat{k}$ and $\vec{B}=3xz\hat{i}+2yz\hat{j}-z^2\hat{k}$, then evaluate $(\vec{A}\times\vec{\nabla})\times\vec{B}$ at the point $(1,-1,2)$. 3+3

5. i) Calculate the horizontal and vertical components of Coriolis force due to the rotation of earth on a particle of mass m, projected horizontally with a velocity $\vec{v}$.

ii) What is Galilean transformation? Prove that Newton's laws of motion be valid in inertial frame. 3+(1+2)

6. i) Let a uniform horizontal heavy beam of circular cross-section clamped at one end and loaded at free end. Calculate the depression of the loaded end.

ii) Define flexural rigidity of a beam. 4+2

7. i) Find the relation between the internal bending moment and geometrical moment of inertia of the section of the beam about the neutral line axis.

ii) Find the rigidity modulus of a wire by the dynamic method. 3+3

GROUP–D

Answer any **four** questions: 10×4=40

8. i) A particle is thrown vertically upwards with a velocity v_0. Assuming the air resistance to be proportional to the velocity of the particle, show that the maximum height attained by it will be given by

$$x_m = \frac{v_0}{k} + \frac{g}{k^2}\log\frac{g}{g+kv_0},$$

where k is a constant.

ii) Expand in a Fourier series the periodic function f(x) with period $2l$ where $f(x)=|x|$ for $-l<x<l$. Hence show that

$$\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\ldots=\frac{\pi^2}{8}.$$

5+(4+1)

9. i) If $\vec{B} = \vec{\nabla}\times\vec{A}$, prove that $\iint_S \vec{B}.\hat{n}\,dS = 0$ for any closed surface S.

ii) State and prove Stokes theorem.

iii) Verify Stokes theorem for $\vec{F} = \left(x^2+y^2\right)\hat{i} - 2xy\hat{j}$ taken around the rectangle bounded by lines $x = \pm a$, y=0, y=b. 2+4+(1+3)

10. i) A reference frame 'a' rotates with respect to another reference frame 'b' with an angular velocity $\vec{\omega}$. If the position, velocity and acceleration of a particle in frame 'a' are represented by $\vec{r}$, $\vec{v}_a$ and $\vec{f}_a$, show that the acceleration of the particle in frame 'b' is given by

$$\vec{f}_b = \vec{f}_a + 2\vec{\omega}\times\vec{v}_a + \vec{\omega}\times\left(\vec{\omega}\times\vec{r}\right) + \frac{d\vec{\omega}}{dt}\times\vec{r}.$$

ii) What do you mean by impulsive force and impulse?

iii) Show that for a system of particles, the torque about any point due to mutually interacting forces, assumed to be central, vanishes. Hence show that for the above system

$\frac{d\vec{L}}{dt} = \vec{N}$. State, from above, the law of conservation of angular momentum.

4+(3+1)+(1+1)

11. i) Define gravitational self-energy. Calculate the gravitational self-energy of a homogeneous sphere of mass M and radius R.

    ii) A particle, acted upon by a central force, describes an orbit given by $r = a(1+\cos\theta)$, 'a' is constant. Show the nature of the force and its graphical representation.

    iii) Define gravitational mass and inertial mass. Are the two masses of the body different? Explain your answer. (1+3)+(3+1)+(1+1)

12. i) Obtain an expression for the length h through which a liquid of surface tension S will rise in a capillary tube of radius r. What is Jurin's law?

    ii) A drop of water 0.5cm in radius is split into 1000 equal droplets. Find the work done and calculate the pressure inside one of the droplets, surface tension of water is 72 dynes.

    iii) Give an account of the molecular theory of surface tension. (3+1)+4+2

13. i) Derive Poiseuille's formula for the steady flow of an incompressible viscous liquid through a horizontal capillary of uniform cross-section. What are the chief corrections to be applied to the formula?

    ii) A horizontal tube of diameter 2mm and length 50cm is connected to the bottom of a cubical tank of side 100cm containing water of viscosity 0.01 poise. The tank is initially full. Water is then allowed to flow out through the tube. Find the time after which the tank will be quarter full (g=980 cm/s^2).

    iii) Obtain, by the method of dimensions, a relation between the critical velocity and Reynold's number for a liquid flowing through a capillary tube. (3+2)+3+2