KANDI RAJ COLLEGE – DEPARTMENT OF MATHEMATICS – INTERNAL EXAMINATION – 5TH SEMESTER

Full marks = 10 + 10 + 10 + 10 **Dated: 14.03.2021**

ANSWER EACH PART IN SEPARATE ANSWER-SCRIPT:

PART – 1 **F.M. = 10**

(1) Form a partial differential equation from, $z = f(x^2 - y^2)$. [2]

(2) Find the partial differential equation of all spheres having center in the xy – plane and radius λ. [2]

(3) Define first order partial differential equation of linear, semi-linear, quasi-linear and non-linear type. [2]

(4) Solve: $y^2p - xyq = x(z - 2y)$. [2]

(5) Show that the equation $z_{xx} - x^2 z_{yy} = 0, (x \neq 0)$ is hyperbolic and find its characteristics. [2]

PART – 2 **F.M. = 10**

(1) Determine the number of elements of order 3 in a non-cyclic group of order 57. [4]

(2) Show that the product of two distinct p-Syllow subgroups of a group G, can never be a subgroup of G. [3]

(3) Prove that the center $Z(G)$ of G is characteristic in G. [3]

PART – 3 **F.M. = 10**

(1) Find the basic feasible solutions of the system of equations: [3]

$$x_1 + 2x_2 + 3x_3 = 6$$
$$2x_1 + x_2 + 4x_3 = 3$$
$$x_1, x_2, x_3 \geq 0$$

(2) Solve by graphical method: [3]

$$Minimize\ z = x_1 + x_2$$
$$Subject\ to\ \ 5x_1 + 9x_2 \leq 45$$
$$x_1 + x_2 \geq 2$$
$$x_1 \leq 4$$
$$x_1, x_2 \geq 0$$

(3) Solve by simplex method: [4]

$$Maximize\ z = 2x_2 + x_3$$
$$Subject\ to\ \ x_1 + x_2 - 2x_3 \leq 7$$
$$-3x_1 + x_2 + 2x_3 \leq 3$$
$$x_1, x_2, x_3 \geq 0$$

PART – 4 **F.M. = 10**

Answer any **TWO (2)** questions from below:

(1) If the joint probability density of X and Y is given by, [5]

$$f(x,y) = \begin{cases} \frac{2}{3}(x + 2y), 0 < x < 1, 0 < y < 1 \\ 0, elsewhere \end{cases}$$

Find the conditional mean and conditional variance of X, given $Y = \frac{1}{2}$.

(2) If a random sample of size $n = 20$ from a normal population with variance $\sigma^2 = 225$, has mean $\bar{X} = 64.3$, construct 95% confidence interval for the population mean μ. Given that $P(Z > 1.96) = 0.025$, where Z is normal $(0,1)$ variate. [5]

(3) If X and Y are standardized random variable, and $r(aX + bY, bX + aY) = \frac{1+2ab}{a^2+b^2}$, find $r(X,Y)$, the correlation coefficient between X and Y. [5]