# U.G. 1st Semester Examination - 2020

# STATISTICS

## [HONOURS]

**Course Code : STAT-H-/CC-T-2**

**(Probability and Probability Distribution-I)**

Full Marks : 50 (40+10) Time : $2\frac{1}{2}$ Hours

*The figures in the right-hand margin indicate marks.*

*Candidates are required to give their answers in their own words as far as practicable.*

*Notations and symbols have their usual meaning.*

1. Answer any **five** questions: 2×5=10

   i) Two biased coins $C_1$ and $C_2$ have probabilities of showing heads (2/3) and (3/4) respectively. If both coins are tossed independently two times each, find the probability of getting exactly two heads out of these four tosses.

   ii) Write down the axiomatic definition of probability.

   iii) Define pairwise independence and mutual independence.



   iv) Define conditional probability. Suppose $A$ and $B$ are events with $P(A) = 0.5$, $P(B) = 0.4$ and $P(A \cap B^c) = 0.2$. Then find $P(B^c|A \cup B)$.

   v) Define cumulative distribution function. Write down its properties.

   vi) What is loss of memory property of a distribution? Name a distribution which possesses this property.

   vii) Define marginal and conditional distribution.

   viii) Find the mean of a negative binomial random variable.

2. Answer any **two** questions: 5×2=10

   i) State the theorem of total probability. Suppose you have one fair coin and one biased coin which lands Head with probability (3/4). You choose one of the coins at random and toss it three times independently. What is the probability that it lands Heads all the three times?

   ii) State and prove Poincare's theorem.

   iii) The joint p.m.f. of two discrete random variables $X_1$ and $X_2$ is

$$p(x_1, x_2) = \binom{n_1}{x_1}\binom{n_2}{x_2 - x_1} p^{x_2} (1-p)^{n_1+n_2-x_2}$$

with $x_1 \le x_2 \le n_2 + x_1$; $0 \le x_1 \le n_1$.

Find the marginal distribution of $X_1$.

iv) Clearly stating the assumptions, derive the Poisson approximation of Binomial distribution.

3. Answer any **two** questions: 10×2=20

i) State Classical definition of probability and clearly mention its limitations. A box contains 4 red, 5 white and 6 black balls. A person draws 4 balls from the box at random. Find the probability that among the balls drawn there is at least one ball of each colour.

ii) State and prove Bayes' theorem. An urn contains n white and m black balls, a second urn contains N white and M black balls. A ball is randomly transferred from the first urn to the second urn and then from the second to the first urn. If a ball is now selected randomly from the first urn, prove that the probability that it is white is



$$\frac{n}{n+m} + \frac{mN - nM}{(n+m)^2 + (N+M+1)}$$

iii) Distinguish between discrete and continuous random variables. Define probability mass function and probability density function. Derive the moment measure of skewness for a *Bin(n,p)* distribution. When does this distribution be symmetric?

iv) a) $X$ and $Y$ are jointly distributed discrete random variables. If $X$ and $Y$ are independent, show that $Cov(X, Y) = 0$. Is the converse true?

b) Write down the pmf of trinomial distribution and derive the marginal distribution of any component.

[ *Internal Assessment : 10* ]

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