

U.G. 6th Semester Examination - 2021

PHYSICS

[HONOURS]

Course Code : PHY-H-CC-T-14

(Statistical Mechanics)

Full Marks : 40

Time : $2\frac{1}{2}$ Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

1. Answer any **five** questions: 2×5=10
- a) What do microstates and macrostates mean for a statistical system of N particles moving in three dimensions?
- b) Suppose 1,000 particles are distributed along a line of length $1m$ with a distribution function $f(x) = Bx(l-x)$. Calculate the number of particles that lie between $0.5m \leq x \leq 0.50lm$. (Particle number may be fractional)
- c) The description of the microcanonical ensemble is given in terms of N , V and the energy between E and $E+\Delta E, \Delta E \ll E$. Why do we need the spread ΔE in energy?

[Turn over]

- d) Define entropy in the microcanonical ensemble.
- e) What is Bose – Einstein condensation?
- f) Show that for a photon gas, the density of states is proportional to $E^{\frac{1}{2}}$ where E corresponds to the energy.
- g) Find the number of ways in which three identical spin $\frac{1}{2}$ fermions can be distributed in two nondegenerate distinct energy levels.
- h) Distinguish between canonical and grand canonical ensembles.

2. Answer any **two** questions : 5×2=10
- a) What are the characteristic features of liquid Helium at low temperature? What is λ transition? Why is it named so? 2+2+1
- b) What is phase space? Draw the phase space for a linear harmonic oscillator of energy E and angular frequency ω . What is the area of the phase space? 1+2+2
- c) What does it mean by “Degeneracy”? Differentiate between Maxwell-Boltzmann, Bose-Einstein, and Fermi-Dirac statistics. 2+3

- d) State the applicability of Saha's ionization formula. Write down its expression explaining each term. 2+3

3. Answer any **two** questions : 10×2=20

- i) a) Write down the expressions for average number of particles in the i_{th} energy level, having energy ϵ_i and degeneracy g_i for Bose and Fermi distributions.

- b) What is the value of the chemical potential for a photon gas in equilibrium and why?

- c) Draw the temperature dependence of Fermi distribution function for $T = 0$ K and $T > 0$ K. Locate the Fermi energy in the diagram.

- d) The density of free electrons in Na is $2.5 \times 10^{28} \text{m}^{-3}$. Calculate the Fermi energy of the free electron gas and compare it with $k_B T$ for $T = 300$ K. 3+2+2+3

- ii) a) Give Planck's quantum postulates.
b) Derive Stefan's law and Wien's displacement law from Planck's law of black body radiation.

- c) What is the wavelength of maximum intensity of radiation radiated from a source at temperature 3000°C ? Wien's constant = 2.898×10^{-3} mK. 2+6+2

- iii) If the partition function of a system of N indistinguishable particles is given by –

$$Z(\beta) = \frac{V^N}{N!} \left(\frac{2\pi m k_B T}{h^2} \right)^{3N/2}$$

Calculate the Helmholtz free energy (F) for the same. Hence calculate the average energy U for the system. Using the thermodynamic relation $F = U - TS$, derive the Sackur-Tedrode equation. How does this equation solve Gibbs paradox? 3+2½+2½+2

- iv) a) Discuss experimental verification of Maxwell's velocity distribution law.
b) What do you understand by Brownian motion?
c) Derive Einstein's formula for the mean square displacement of a Brownian particle. 3+2+5