

**U.G. 4th Semester Examination - 2021**

**PHYSICS**

**[HONOURS]**

**Course Code : PHY-H-CC-T-8**

Full Marks : 20

Time : 1 Hour

*The figures in the right-hand margin indicate marks.*

*Candidates are required to give their answers in their own words as far as practicable.*

**GROUP-A**

1. Answer any **five** questions: 1×5=5
- a) Find the Laplace transform of the function  $\delta(t-t_0)$ .
- b) If  $g(\omega)$  is Fourier transform of  $f(x)$ , show that  $g(-\omega) = g^*(\omega)$  is a necessary and sufficient condition for  $f(x)$  to be real.
- c) If  $f(z) = 1 + iz$ , is  $\overline{f(z)} = f(\bar{z})$ ?
- d) Evaluate  $\tanh\left(\frac{i\pi}{4}\right)$ .
- e) Find the roots of  $\sqrt[3]{i}$ .
- f) If  $f(s)$  is the Laplace transform of  $f(t)$  then find the Laplace transform of the function  $f(at)$ .

[Turn over]

- g) State whether the function  $|z|^2$  satisfy the Cauchy-Riemann condition.
- h) Find the Fourier transform of  $\delta(t)$ .

**GROUP-B**

2. Answer any **one** questions from the following:

5×1=5

- a) Find the Fourier transform of the finite wave train

$$f(t) = \begin{cases} \cos \omega_0 t & |t| \leq a \\ 0 & |t| > a \end{cases}$$

- b) Find the inverse Laplace transform of

$$f(s) = \frac{6}{(s^2 + 9)^2}$$

- c) Find the Cauchy-Riemann equations in polar coordinates.

**GROUP-C**

Answer any **one** question from the following: 10×1=10

3. a) Find the indefinite integral

$$\int_{-\infty}^{\infty} \frac{dx}{x^2 + a^2}$$

by contour integration.

5+5

b) Evaluate the integral

$$\oint_C \frac{\sin 2z dz}{(6z - \pi)^3}$$

where C is the circle  $|z| = 3$ .

4. a) Solve the one-dimensional heat flow equation

$$\frac{\partial \psi}{\partial t} = \kappa^2 \frac{\partial^2 \psi}{\partial x^2}$$

using Fourier transform where the solution  $\psi(x, t)$  is the temperature at position x and time t.

b) Find the Fourier transform of the normalised Gaussian distribution

$$f(t) = \frac{1}{a\sqrt{2\pi}} \exp\left(-\frac{t^2}{2a^2}\right), \quad -\infty < t < \infty. \quad 5+5$$

5. a) Find the inverse Laplace transform of the function

$$f(s) = \frac{s^2}{(s^2 + a^2)(s^2 + b^2)}$$

for  $a^2 \neq b^2$ .

b) Solve the initial value problem

$$\frac{d^2 y}{dt^2} + 9y = 2 \sin 3t,$$

using Laplace transform when  $y(0) = 1$  and  $y'(0) = 0$ . 5+5

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