Use Separate Answer-scripts for Different Papers

CC – T – 08 **10**

Answer any **TWO (2)** questions: 2×5

1. Prove that, if a function $f:[a,b] \to \mathbb{R}$ be integrable on $[a,b]$, then f^2 is also integrable on $[a,b]$. 5

2. Define uniform convergence of a sequence of functions $\{f_n\}$ on an interval **I**. 5
Examine the uniform convergence of the sequence of functions $\{f_n\}$ on [0, 1];
where for each $n \in \mathbb{N}, f_n(x) = \frac{x}{1+nx^2}, x \in [0,1]$

3.(i) Find the radius of convergence of the power series 2
$\sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{n+1}(x+1)^n$.

(ii) Let $\sum_{n=0}^{\infty} a_n x^n$ be a power series with radius of convergence R(>0) and $f(x)$ be the sum of the series on (-R, R). Then prove that $f^k(0) = k!\, a_k \ (k = 0, 1, 2, \ldots)$. 3

CC – T – 09 **10**

Answer any **TWO (2)** questions: 2×5

1. Find the shortest distance from the point $(0, b)$ on the Y-axis to the parabola $x^2 - 4y = 0$. Use Lagrange's method.

2. Change the order of integration to evaluate

$$\int_0^a \left\{ \int_{\frac{x^2}{2}}^{2a-x} xy\ dy \right\} dx$$

3. Verify Stokes theorem for the function $\vec{F} = x^2\vec{\imath} - xy\vec{\jmath}$ integrated round the square in the plane $z = 0$ and bounded by the lines $x = 0,\ y = 0,\ x = a,\ y = a$.

CC – T – 10 **10**

Answer any **TWO (2)** questions: 2×5

1. Let $M_2(\mathbb{Z}) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} : a, b, c, d \in \mathbb{Z} \right\}$. Then $M_2(\mathbb{Z})$ forms a ring with respect to matrix addition " + " and matrix multiplication " × ".
Let $S = \left\{ \begin{pmatrix} a & 0 \\ b & 0 \end{pmatrix} : a,\ b \in \mathbb{Z} \right\}$. Show that S is a subring of $M_2(\mathbb{Z})$ but not an ideal of $M_2(\mathbb{Z})$.

2. Let $C = \mathbb{R} \times \mathbb{R} = \{(a,b) : a, b \in \mathbb{R}\}$. Then show that $\langle C, +, \cdot \rangle$ is a field where " + " and " · " are defined by $(a,b) + (c,d) = (a+c, b+d)$ and $(a,b)\cdot(c,d) = (ac - bd,\ bc + ad)$

3. Prove that an ideal M in a ring of integers $\langle \mathbb{Z}, +, \cdot \rangle$ is maximal if and only if $M = p\mathbb{Z}$, where p is prime.

SEC – T – 2A **05**

Answer any **ONE (1)** question: 1×5

1.(i) Does there exist a graph with 6 edges and degree sequence (1,1,2,4,5,5)? 2

(ii) Define a simple graph. [1+2]
Examine if a simple graph with degree sequence (2,2,4,5,5) exists.

2. Let G be a connected graph of order $n \geq 3$ and size m. Then show that G is Hamiltonian if $m \geq \frac{1}{2}(n-1)(n-2) + 2$. [3+2]
Is the converse true? Give an example to illustrate.

For Mathematics Honours students the question ENDS

Students other than Mathematics Honours: Go to Next Page

GE – T – 04 [For students other than Mathematics Honours] **10**

Answer **all** the questions

1. Solve $(x^3 + 3y^2x)dx + (y^3 + 3x^2y\,)dy \;= 0.$ 2
2. Find the complete primitive of the differential equation 2
   $y = px + f(p)$ where $p = \frac{dy}{dx}$.
3. Solve the differential equation $\frac{d^3y}{dx^3} - 2\frac{d^2y}{dx^2} - \frac{dy}{dx} + 2y = 0.$ 2
4. Solve the differential equation $x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = \; log\, x.$ 2
5. Give one example of each of the following exclusive types of 1st order partial differential equation in two independent variables: 2
   (i) Linear
   (ii) Semilinear
   (iii) Quasilinear
   (iv) Nonlinear.

For Other Honours students the question ENDS