Kandi Raj College – Department of Mathematics – Internal Examination – 4th Semester – Programme course

Full marks: G-CC-T-04 = 10
For students opting for Mathematics as SEC: G-SEC-T-2A = 05

	G-CC – T – 04 <u>Answer any **TWO (2)** questions:</u>	**10** 2×5
1.	Show that the necessary and sufficient condition that a non-empty subset H of a finite group $\langle G,*\rangle$ to be a subgroup of G is that $a \in H, b \in H \Rightarrow a * b \in H$. Give a suitable example to show that this does not hold for an infinite group.	3 + 2
2.	Use Lagrange's theorem to prove the following: (a) The order of every element of a finite group must divide the order of the group. (b) For any element a of a finite group $G, a^{O(G)} = e$, where e is the identity.	3+2
3.	Let $M_2(\mathbb{C}) = \left\{\begin{pmatrix} a & b \\ -\bar{b} & \bar{a} \end{pmatrix} : a, b \in \mathbb{C}, \bar{a}, \bar{b}\ are\ conjugate\ of\ a, b\ respectively\right\}$ Show that $\langle M_2(\mathbb{C}), +, \cdot\rangle$ forms a ring with respect to matrix addition " $+$ " and matrix multiplication " $\cdot$ ". Is it a ring with unity? Is it a commutative ring?	3+1+1

End of Question for **G-CC – T – 04**

*The question below is **Only for students opting for Mathematics as SEC***

Use separate answer-script to answer and submit separately

	G-SEC – T – 2A <u>Answer any **ONE (1)** question:</u>	**05** 1×5
1.	State and prove *Handshaking lemma*. Use it to show that there does not exist a graph with degree-sequence (1, 2, 3, 4, 5).	1+2+2
2.	Define a complete graph K_n. Show that every complete graph K_n must have exactly $\frac{1}{2}n(n-1)$ edges. Use it to show that the size of K_n is a multiple of n, if n is odd.	1+2+2

End of Question for **G-SEC – T – 2A**