

U.G. 2nd Semester Examination - 2021

STATISTICS

[HONOURS]

Course Code : STAT-H-CC-T-04

(Probability and Probability Distributions-II)

Full Marks : 25(20+5)

Time : 1 Hour

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

Notations and symbols have their usual meaning.

1. Answer any **five** questions: 1×5=5
- If the joint pdf of two random variables X and Y be $f(x,y)$, write down an expression for the conditional pdf of Y|X.
 - Write down Chebyshev's inequality.
 - Write down the pdf of a logistic distribution.
 - If (X, Y) assumes only two pairs of values $(-1, -5)$ and $(1, 10)$, what is the value of ρ ?
 - Mention a situation where Binomial distribution is appropriate to model the data.

- Suppose $X \sim \text{Bin}(n, p)$, $0 < p < 1$. What is the pmf of $Y = n - X$?
- Write down the expression for the mgf of Poisson distribution with parameter λ .
- If (X, Y) follows bivariate normal distribution with parameters $(0, 0, 1, 1, \rho)$, what is the conditional distribution of Y given $X = x$?

2. Answer any **one** question: 5×1=5

- Define p.g.f. of a discrete probability distribution. Derive the p.g.f. of a binomial distribution, $\text{Bin}(n, p)$. Hence find the probability of the value 1.
- Show that:

$$\text{Var}(Y) = \text{Var}\{E(Y | X)\} + E\{\text{Var}(Y | X)\}.$$

3. Answer any **one** question: 10×1=10

- Verify whether the function

$$F(x) = 0 \text{ if } x \leq 1 \quad \text{and } 1-x \text{ if } x > 1$$
 represents a c.d.f. 5
 - If X has the p.d.f. $f(x) = 1/(2\sqrt{3})$ for $-\sqrt{3} \leq x \leq \sqrt{3}$; and zero elsewhere, find an upper bound to $P(|X| \geq 3/2)$. 5

b) i) Write down the p.m.f. of a Hypergeometric distribution and derive its mean. 5

ii) Let X be a non-negative random variable with distribution function F. Show that

$$E(X) = \int_0^{\infty} [1 - F(x)] dx \quad 5$$

[Internal Assessment: 5]
