2021
STATISTICS
[GENERAL]
Paper : I

Full Marks : 100 Time : 3 Hours

The figures in the right-hand margin indicate marks. Candidates are required to give their answers in their own words as far as practicable.

Answer all the questions.

1. Answer any **eight** questions: 1×8=8

   a) True or false: Any non-negative bounded function can be considered as a distribution function of a random variable.

   b) True or false: X, Y ~ Poisson(λ) independently then $Z = \frac{x+y}{2} \sim$ Poisson(λ).

   c) Write the *pdf* of $N(\mu, \sigma^2)$ distribution.

   d) For any two functions u_x and V_x show that $\Delta(u_x V_x) = u_{x+h}\Delta V_x + V_x \Delta u_x$.

   e) What is the appropriate graphical representation of a time series data?

   f) Write the classical definition of probability.

   g) Define expectation of a random variable.

   h) What is frequency curve?

   i) Define correlation coefficient.

   j) What is regression?

2. Answer any **ten** questions: 2×10=20

   a) Define bivariate moments (raw and central).

   b) For two regression lines, y on x and x on y, show that $b_{xy} \times b_{yx} = r^2$.

   c) What is interpolation?

   d) What do you mean by numerical integration?

   e) For any function $f(x)$, find $(1+\Delta)^k f(x)$, where k is an integer.

   f) What is wholesale price index number?

   g) Discuss, through an example, the seasonal variation in time series data.

   h) Define independent events. Give an example of two events which are independent.

   i) Define random variable. Give an example of it.

j) Let X be a random variable with pdf $f(x)=\frac{nx^{n-1}}{\theta^n}; 0<x<\theta$ with $\theta>0$. Find $E(X^r)$ for any positive integer r.

k) If $X \sim N(2,2)$ and $Y \sim N(1,1)$ with X and Y are independent then what is the distribution of $Z = X - Y$.

l) Suppose $X \sim Bin(n, p)$, $0 < p < 1$. For some $t \in R$, find $E\left(t^x\right)$.

m) Discuss the use of multiple bar diagram with an example.

n) Describe, with diagram, the relative position of mean, median and mode for positively skewed distribution.

3. Answer any **seven** questions: 6×7=42

a) A die is rolled five times. Let X be the sum of the face values. Express the events $\{X = 4\}$, $\{X = 6\}$ and $\{X = 30\}$ in terms of elementary events. 6

b) Write Lagrange's interpolation formula and show that the sum of the Lagrangian coefficients is unity. 2+4

c) Describe the iteration process for finding solution of numerical equation. Justify the condition of its convergence. 4+2

d) What is binary commodity? Discuss how the commodities are chosen while constructing wholesale price index number. 1+5

e) Describe the procedure of fitting an exponential trend equation of the form $y_t = ab^t$ to a time series data. 6

f) Show that the correlation coefficient between two variables lies between -1 and +1. Discuss the extreme situations. 5+1

g) Derive the formula for Spearman's rank correlation for untied case. 6

h) Describe different parts of a table for representing statistical data. 6

i) Define conditional probability. Let X_n be the number of heads obtained in first n trials of tossing a perfect coin. Find $P\left(X_n = x \mid X_{n-1} = y\right)$ where $x \geq y$. 2+4

j) Find the measures of skewness and kurtosis for Poisson(λ) distribution and hence comment. 6

4. Answer any **three** questions: 10×3=30

a) i) Using classical definition of probability show that for any two events A and B corresponding to the sample space Ω, if A is a proper subset of B then P(A)< P(B).

ii) For a set of events $\{A_1, A_2, \ldots, A_n\}$ corresponding to the sample space Ω such that

$$P(A_k) \geq 1-\alpha_k, \quad k=1,2,\ldots,n.$$

Show that

$$P(A_1 \cap A_2 \cap \ldots \cap A_n) \geq 1-(\alpha_1+\alpha_2+\ldots+\alpha_n)$$

3+7

b) Describe, in brief, different steps associated with the construction of consumer price index number. 10

c) Describe different components of a time series. Discuss the method of moving average for finding the trend component. 4+6

d) i) Let $F_1(x)$ and $F_2(x)$ be two distribution functions. Define a function $F(x)=\alpha F_1(x)+(1-\alpha)F_2(x), \alpha \in (0,1)$. Check whether F(x) can be considered as a distribution function.

ii) Define :

$$f(x) = \frac{d}{x^3}, \text{ if } x \geq 1$$
$$0 \quad \text{otherwise}$$

Find the value of d such that $f(x)$ is a pdf and then find the expectation of the distribution. 4+(3+3)

e) i) Find the error in Simpson's 1/3rd rule for numerical integration.

ii) Prove that $\int_{3/2}^{5/2} u_x dx = \frac{1}{24}(u_1+22u_2+u_3)$ assuming a suitable polynomial function u_x. 5+5