2021
MATHEMATICS
[GENERAL]
Paper : III

Full Marks : 100 Time : 3 Hours

The figures in the right-hand margin indicate marks.
Symbols have their usual meanings.

GROUP–A
(Linear Programming and Game Theory)
[Marks : 50]

1. Answer any **four** questions: 1×4=4

a) Do the vectors (4, 3, 2), (2, 1, 4), (2, 3, –8) form a basis for E^3?

b) Define convex hull. Give an example.

c) Find the extreme points, if any of the set

$$X = \left\{(x, y)/x^2 + y^2 \le 25\right\}.$$

d) Define feasible solution to a L.P.P.

e) Define convex polyhedron.

[Turn over]

f) Write down the mathematical form of general L.P.P.

2. Answer any **six** questions: 2×6=12

a) Reduce the following problem to standard maximinization form:

$$\text{Maximize } Z = 2x_1 + x_2$$
$$\text{subject to } x_1 \le 4$$
$$2x_1 + x_2 \ge 1, \ x_1, x_2 \ge 0$$

b) Find the dual of the following L.P.P.:

$$\text{Maximize } Z = -x_1 + 3x_2$$
$$\text{subject to } 2x_1 + x_2 \le 1$$
$$3x_1 + 4x_2 \le 5$$
$$x_1 + 6x_2 \le 9$$
$$x_1, \ x_2 \ge 0$$

c) Write the rule for determining saddle point.

d) Using maximini-minimax principle solve the following game:

$$A\begin{bmatrix} 1 & 1 \\ 4 & -3 \end{bmatrix}$$

e) Express (2, 4, –3) as a linear combination of (1, 3, 1) and (0, 2, 5).

f) Find the solution of the equations:

$$2x_1+3x_2+x_3=8$$

$$x_1+2x_2+2x_3=5$$

g) Verify whether the set of vectors form a spanning set for E^3;

$$(1,\ -1,\ 0),\ (0,\ 0,\ 1),\ (1,\ 1,\ 0).$$

h) What is the convex hull of the set

$$X=\left\{(x,\ y)\Big/\frac{x^2}{3}+\frac{y^2}{2}=1\right\}?$$

3. Answer any **four** questions: 6×4=24

a) Find the optimal solution of the following L.P.P. solving its dual:

$$\text{Minimize } Z=4x_1+3x_2+6x_3$$

$$\text{subject to } x_1+x_3 \geq 2$$

$$x_2+x_3 \geq 5$$

$$x_1,\ x_2,\ x_3 \geq 0$$

b) Find the optimal solution of the following transportation problem:

	D_1	D_2	D_3	a_i
O_1	10	9	8	8
O_2	10	7	10	7
O_3	11	9	7	9
O_4	12	14	10	4
b_j	10	10	8	

c) Use dominance property to reduce the following pay-off matrix and solve the game:

$$\begin{array}{cc} & \text{Player B} \\ \text{Player A} & \begin{array}{c|cccc|} & B_1 & B_2 & B_3 & B_4 \\ A_1 & -5 & 3 & 1 & 15 \\ A_2 & 5 & 5 & 4 & 6 \\ A_3 & -4 & -2 & 0 & -5 \end{array} \end{array}$$

d) Find a B.F.S. of the following system of equations:

$$x_1+4x_2-x_3=3$$

$$5x_1+2x_2+3x_3=4$$

e) Solve the following L.P.P. graphically:

Minimize $Z = x_1 + 7x_2$

subject to $-x_1 + 2x_2 \le 8$

$x_1 - x_2 \le 4$

$x_1,\ x_2 \ge 0$

f) Solve by simplex method:

Maximize $Z = 5x_1 - 2x_2 + 3x_3$

subject to $2x_1 + 2x_2 - x_3 \ge 2$

$3x_1 - 4x_2 \le 3$

$x_2 + 3x_3 \le 5$

$x_1,\ x_2,\ x_3 \ge 0$

4. Answer any **one** question: 10×1=10

a) Transform to LPP and hence solve the game problem whose pay-off matrix is

$$\begin{bmatrix} 2 & -3 & 4 \\ -3 & 4 & -5 \\ 4 & -5 & 6 \end{bmatrix}$$

b) Solve the following transportation problem:

	D_1	D_2	D_3	a_i
O_1	50	30	220	1
O_2	90	45	170	3
O_3	250	200	50	4
b_j	4	2	2	

GROUP–B

(Probability Theory)

[Marks : 30]

5. Answer any **four** questions: 1×4=4

a) Give the definition of k-th central moment of a random variable X.

b) Define the variance of a random variable.

c) Give examples of two continuous probability distributions.

d) Give the classical definition of probability.

e) Define conditional probability.

f) State the Bayes' theorem on conditional probability.

6. Answer any **four** questions: 2×4=8

a) Show that

$$P(X.Y) > 0 \Rightarrow E(XY) > E(X)E(Y).$$

b) Prove that the probability distribution function is rightly continuous.

c) Find the parameters of binomial variate X, whose mean and variance are $\frac{15}{2}, \frac{15}{4}$.

d) If A, B, C are any three events, then prove that

$$P(A+B+C)=P(A)+P(B)+P(C)-P(AB)-P(BC)-P(CA)+P(ABC).$$

e) In any random experiment E, if A and B are any two events, then show that

$$P(AB)=P(A).P(B/A)=P(B).P(A/B).$$

f) Two dice are thrown simultaneously. Find the probability of getting a total of 4 points in a single throw.

7. Answer any **three** questions: 6×3=18

a) Find k such that

$$f(x)=\begin{cases} 0 & \text{if} \quad x<0 \\ kx(1-x) & \text{if} \quad 0\le x<1 \\ kx^2 & \text{if} \quad 1\le x\le 2 \\ 0 & \text{if} \quad x>2 \end{cases}$$

is a probability distribution function. Also obtain the distribution function.

b) Find the n-th moment of the normal distribution N(m.m) about the mean m. $\left[\text{Given that } \Gamma\left(\frac{1}{2}\right)=\sqrt{\pi}\right]$.

c) i) Show that if X and Y are independent then $E(XY)=E(X).E(Y)$ for continuous distribution.

ii) Define mean, covariance and correlation coefficient for two-dimensional random variables X and Y.

d) The probability of a man hitting a target is $\frac{1}{3}$. How many times must he fire so that the probability of hitting the target atleast once is more than 90%?

e) Deduce Poisson distribution from Binomial distribution. Hence obtain mean of Poisson distribution.

GROUP–C

(Statistics)

[Marks : 20]

8. Answer any **four** questions: 1×4=4

   a) What is a Quartile deviation?

   b) Define 'skewness' of a distribution.

   c) Show that the mean deviation about mean is always zero.

   d) Define a Ogive Curve.

   e) Define the median of a distribution.

   f) What is the relation between mean, median and mode in case of a symmetrical distribution?

9. Answer any **three** questions: 2×3=6

   a) Calculate the variance and sd of {3, 4, 8}.

   b) Define Quartiles of a distribution.

   c) State two important properties of regression co-efficient.

   d) For the given set of data {3, 2, 5, 7}, show that G.M.>H.M.

   e) Show that the sum of deviation of the sample $x_1, x_2, ..., x_n$ of size n is zero.

10. Answer any **two** questions: 5×2=10

    a) Show that the A.M. of two regression coefficients is always greater than or equal to the correlation coefficient.

    b) Find the correlation coefficient from the following data:

x	2	3	4	5	6
y	5	2	3	4	1

    c) Draw a pie chart of the following data:

Year	1998	1999	2000	2001	2002
No. of tourist at a place (in thousand)	14	17	20	22	29

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