U.G. 5th Semester Examination - 2020

PHYSICS

[HONOURS]

Course Code : PHY-H-CC-T-11

(Quantum Mechanics & Applications)

Full Marks : 40 Time : $2\frac{1}{2}$ Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

1. Answer any **five** questions: 2×5=10

   a) Show that The group velocity of the wave packet is equal to the classical velocity of the particle.

   b) What is operator in quantum mechanics? Explain why 'log', or 'sin' is not a quantum mechanical operator.

   c) Define orthogonal and orthonormal state using Dirac bra-ket notation.

   d) Show that if ψ be an eigen function of the operator A with eigen value λ, then it is also eigenfunction of e^A with eigenvalue e^{λ}.

   e) Using Heisenberg uncertainty relation show that electron cannot exist in the nucleus.

   f) What is the physical interpretation of a wave function? Write its orthogonal and normalized conditions.

   g) Check whether the operator $\hat{O} = -i\hbar\frac{d}{dx}$ is linear or not.

   h) Find out the wave function of a free particle in Cartesian coordinate.

2. Answer any **two** questions: 5×2=10

   a) i) The time independent wave function of a particle of mass *m* moving in a potential $V(x) = \alpha^2 x^2$ is

$$\psi(x) = \exp\left(-\sqrt{\frac{m\alpha^2}{2\hbar^2}}x^2\right)$$

   α being a constant. Find the energy of the system.

   ii) Give a physical example of particle in a box. 1+4

   b) i) Write down the ground state wave function of a harmonic oscillator.

ii) Interpret probability current densities of Wave Function in three dimensions. 1+4

c) i) Derive Time dependent Schrodinger equation for matter waves.

ii) Write down the expression of angular momentum operator in spherical polar coordinate. 3+2

d) i) What is the momentum representation of the position operator $\hat{x}$?

ii) Show that this representation satisfies the position-momentum commutation relation.

iii) Write down the radial wave function of the 1st and excited state of Hydrogen atom and plot the function. 1+2+2

3. Answer any **two** questions: 10×2=20

a) i) Consider a particle whose normalized wave function is

$$\psi(x) = \begin{cases} 2\alpha\sqrt{\alpha}xe^{-\alpha x} & x>0 \\ 0 & x<0. \end{cases}$$

Calculate $\langle x\rangle$ and $\langle x^2\rangle$.

ii) The mathematical representation of a spherical wave travelling outwards from a point is given by

$$\psi(r) = \frac{A}{r}e^{ikr}$$

where A is a constant and k is the wave vector. Evaluate its probability current density.

iii) Show that E must exceed the minimum value of V(x) for every normalizable solution to the time –independent Schrodinger equation. 4+4+2

b) i) Derive The Schrodinger Equation for the Harmonic Oscillator.

ii) Using the uncertainty principles between x and p derive the zero-point energy:

$E_0 = \frac{1}{2}\hbar\omega$ for a harmonic oscillator with natural frequency ω.

iii) What is the difference between Zeeman effect and Stark effect? 4+4+2

c) i) What is space quantization? Explain with a suitable diagram. Discuss briefly spin angular momentum. What is the importance of Stern-Gerlach experiment?

ii) Discuss normal Zeeman effect with energy level diagram. (2+2+1)+5

d) i) Show that there is no splitting in the energy level of S orbital of Sodium.

ii) You have a system of two electrons whose orbital quantum numbers are $I_1 = 2$ and $I_2 = 4$ respectively.

A) Find the possible values of l (total orbital angular momentum quantum number) for the system.

B) Find the possible values of s (total spin angular momentum quantum number) for the system.

C) Find the possible values of j (total angular momentum quantum number) for the system.

iii) Define symmetric and antisymmetric wave function. Determine whether the following function is symmetric or antisymmetric.

$$\phi(x_1, x_2) = \frac{3(x_1 - x_2)}{2(x_1 - x_2)^2 + 7}$$

iv) Consider a system of N identical particles. What would happen to the probability density if the particles interchange their position? 2+3+(1+1)+3

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