U.G. 5th Semester Examination - 2020

MATHEMATICS

[HONOURS]

Discipline Specific Elective (DSE)

Course Code : MATH-H-DSE-T-2A

Full Marks : 60 Time : $2\frac{1}{2}$ Hours

The figures in the right-hand margin indicate marks. Candidates are required to give their answers in their own words as far as practicable.

1. Answer any **ten** questions: 2×10=20

a) Find the constant α such that the function

$$f(x) = \begin{cases} \alpha x^2 & 2 < x < 3 \\ 0 & otherwise \end{cases}$$

is a density function, and compute $P(1 < X < 2)$.

b) If $X^*=(X-\mu)/\sigma$ is a standardized random variable, prove that (i) $E(X^*) = 0$, (ii) $Var(X^*) = 1$.

c) The quantity $E[(X-a)^2]$ is minimum when $a = \mu = E(X)$.

d) Define coefficient of skewness and kurtosis of a distribution.

e) State Chebyshev's inequality for a continuous random variable.

f) Find the expectation of the sum of points in tossing a pair of fair dice.

g) Find the expectation of a discrete random variable x whose probability function is given by $f(x) = (1/3)^x \quad (x = 1,2,3\ ...)$

h) Find the probability of not getting a 7 or 11 total on either of two tosses of a pair of fair dice.

i) Can the function

$$F(X) = \begin{cases} c(1-x^2) & 0 \le x \le 1 \\ 0 & otherwise \end{cases}$$

be a distribution function? Explain.

j) Prove that $-1 \le \rho \le 1$.

k) Let X have density function $f(x) = \begin{cases} 1/(b-a) & a \le x \le b \\ 0 & otherwise \end{cases}$. Find the kth moment about

i) the origin,

ii) the mean.

l) Find (i) the covariance , (ii) correlation coefficient of two random variables X and Y if

$$E(X) = 2, E(Y) = 3, E(XY) = 10, E(X^2) = 9, E(Y^2) = 16.$$

m) Define conditional expectation.

n) Find the probability of drawing three aces at random from a deck of 52 ordinary cards if the cards are (i) replaced and (ii) not replaced.

o) Find the characteristic function of a random variable X having density function $f(x) = ce^{-k|x|}$, $-\infty < x < \infty$, where $k > 0$ and c is suitable constant.

2. Answer any **four** questions: 5×4=20

a) State and prove law of large number theorem. 5

b) Find the variance and standard deviation of the sum obtained by tossing a pair of fair dice. 3+2

c) Show that $E[(X-\mu)^2] = E(X^2) - [E(X)]^2$. Hence find var(X) and σ_x, where $E(X) = 2, E(X^2) = 8$. 2+3

d) If X and Y are independent random variables, then show that $E(XY) = E(X)E(Y)$. 5

e) Let X have density function

$f(x) = \begin{cases} e^{-x} & x > 0 \\ 0 & x \le 0 \end{cases}$. Find the density function of $Y = X^2$. 5

f) Define type-I and type-II errors.

The probability density function of the random variable X is

$f(x) = \begin{cases} \frac{1}{\lambda} e^{-x/\lambda} & x > 0 \\ 0 & x \le 0 \end{cases}$, where $\lambda > 0$. For testing the hypothesis $H_0 : \lambda = 3$ against $H_A : \lambda = 5$ a test is given as "Reject H_0 if $X \ge 4.5$". Find the probability of type-I error and power of the test. 2+3

3. Answer any **two** questions: 10×2=20

a) Design a decision rule to test the hypothesis that a coin is fair if a sample of 64 tosses of the coin is taken and if a level of significance of (a) 0.05, (b) 0.01 is used. How could you design a decision rule to avoid a type-II error? 5+5

b) Prove that the mean and variance of binomially distributed random variable are respectively, $\mu = np \text{ and } \sigma^2 = npq$. If the probability of defective bolt is 0.25, find the mean and standard deviation for the number of defective bolts in a total of 400 bolts. 5+5

c) Let X and Y be independent random variables having density function

$f(u) = \begin{cases} 2e^{-2u} & u \ge 0 \\ 0 & otherwise \end{cases}$.

Find $E(X+Y), E(X^2+Y^2)$ and $E(XY)$.

Does $(i) E(X+Y) = E(X) + E(Y)$,

(ii) $E(XY) = E(X)E(Y)$? Explain. 5+5

d) If the random variable X and Y have the joint density function

$f(x,y) = \begin{cases} xy/96 & 0 < x < 4, 1 < y < 5 \\ 0 & otherwise \end{cases}$, find the density function of $U = X + 2Y$. 10