B.Sc. 5th Semester (General) Examination, 2020 (CBCS)

Subject: MATHEMATICS

Paper: MATH-G-DSE-T-01-A

(Matrices and Linear Algebra)

Full Marks: 60 **Time: 3 Hours**

The figures in the right-hand margin indicate marks.

The notations and symbols have their usual meanings.

1. Answer any **TEN** questions. $\mathbf{2 \times 10 = 20}$

i) If $A = \begin{pmatrix} 2 & -5 \\ 3 & 1 \end{pmatrix}$, find scalars a, b such that $I + aA + bA^2 = 0$.

ii) Find the matrices A and B such that $2A + B^t = \begin{pmatrix} 2 & 5 \\ 10 & 2 \end{pmatrix}$ and $A^t + 2B = \begin{pmatrix} 1 & 8 \\ 4 & 1 \end{pmatrix}$.

iii) Determine the value of k so that $\{(1,2,-1),(2,0,1),(-1,1,k)\}$ represents a basis of $\mathbb{R}^3$.

iv) Express α in terms of β and γ, where $\alpha = (3,7), \beta = (2,4), \gamma = (-1,1)$.

v) Determine the value of λ so that the matrix $\begin{pmatrix} 0 & 1 & -2 \\ 1 & \lambda & 3 \\ 2 & 1 & 2 \end{pmatrix}$ is non-singular.

vi) Test whether the mapping $T: \mathbb{R}^2 \to \mathbb{R}^2$ defined by $T(x, y) = (2x + 1, 2y - 1)$ represents a linear transformation or not.

vii) Determine the characteristic polynomial of $A = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 5 & 2 \\ 0 & 0 & -2 \end{pmatrix}$, and hence find the eigen values of A.

viii) Let $A = \begin{pmatrix} 1 & -1 \\ 0 & 2 \end{pmatrix}$, then find eigen values of A^5.

ix) Let $W = \{(x, y, z) \in \mathbb{R}^3 : x - 4y + 3z = 0\}$, then check whether W is a subspace of $\mathbb{R}^3$.

x) Find inverse of $\begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix}$.

xi) Determine eigen vectors of $\begin{pmatrix} 1 & 1 \\ 1 & 3 \end{pmatrix}$.

xii) For a matrix A, if $A^2 = A$, then find the eigen values of A.

xiii) Express $A = \begin{pmatrix} 1 & 2 \\ 2 & 5 \end{pmatrix}$ as the product of elementary matrices.

xiv) Is there a linear transformation $T: \mathbb{R}^3 \to \mathbb{R}^2$ such that $T(1,0,3) = (1,1)$ and $T(-2,0,-6) = (2,1)$? Justify.

xv) Let $T: \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation defined by $T(a_1, a_2) = (a_1 + a_2, a_1)$. Is T bijective? Justify.

2. Answer any **<u>FOUR</u>** questions. **$5 \times 4 = 20$**

a) If $(I_n - A)(I_n + A)^{-1}$ is orthogonal, then show that A is skew symmetric.

b) Let $W = \{(x,y,z) \in \mathbb{R}^3: 2x - y + 3z = 0, x + y + z = 0\}$. Show that W is a basis of $\mathbb{R}^3$.

c) Using elementary operations, find the inverse of $A = \begin{pmatrix} 1 & 1 & 2 \\ 1 & 2 & -2 \\ -2 & -4 & -4 \end{pmatrix}$.

d) Determine γ and μ so that the system of linear equations $x + y + z = 1, x + \gamma y + 4z = \mu$ and $x + \gamma^2 y + 10z = \mu^2$ has i) unique solution, ii) no solution and iii) an infinite number of solutions.

e) Show that $W_1 = \{(x,y,z): x + y + z = 0\}$ and $W_2 = \{(x,y,z): y = z\}$ are two subspaces of $\mathbb{R}^3$. Find dimension of $W_1, W_2$and $W_1 \cap W_2$.

f) Show that the mapping $T: \mathbb{R}^2 \to \mathbb{R}^3$ defined by $T(a_1, a_2) = (2a_1 - a_2, 3a_1 + 4a_2, a_1)$ represents a linear transformation. Hence find the matrix representation of T with respect to the standard ordered bases of $\mathbb{R}^2$ and $\mathbb{R}^3$. **2+3**

3. Answer any **<u>TWO</u>** questions. **10$\times$ 2=20**

A) i. Obtain a basis of $\mathbb{R}^3$ containing the vectors $(2,-1,0)$ and $(1,3,2)$. **4+6**

ii. Determine the linear transformation $T: \mathbb{R}^3 \to \mathbb{R}^3$ that maps the basis $\{(0,1,1), (1,0,1), (1,1,0)\}$ of $\mathbb{R}^3$ to $(2,1,1), (1,2,1), (1,1,2)$, respectively. Find $Ker\, T$, $Im\, T$. Also, verify rank-nullity theorem for T.

B) i. Prove that every non-null subspace W of a finite dimensional linear space $V(F)$ is finite dimensional and $dim\, W \leq dim\, V$. **4+6**

ii. Diagonalize the matrix $A = \begin{pmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{pmatrix}$.

C) i. Let V and U be two finite dimensional vector spaces over a same scalar field F. Then prove that $L(V,U)$ is finite dimensional. Also find its dimension in terms of the dimensions of V and U. **5+(1+4)**

ii. Define isomorphism on a vector space. A linear mapping $T: \mathbb{R}^3 \to \mathbb{R}^3$ maps the vectors $(1,2,3), (3,0,1)$ and $(0,3,1)$ to $(-3,0,2), (-5,2,-2)$ and $(4,-1,1)$, respectively. Check whether T is an isomorphism.

D) i. Solve the system of linear equations, $x_1 + 2x_2 + 3x_3 = 1, 4x_1 + 5x_2 + 6x_3 = 2$, $7x_1 + 8x_2 + 2x_3 = 3$. **4+3+3**

ii. If μ be an eigen value of an invertible matrix A, then prove that $\frac{1}{\mu}$ is an eigen value of A^{-1}.

iii. Find the matrix representation of the linear transformation $T: \mathbb{R}^3 \rightarrow \mathbb{R}^2$ defined by $T(x, y, z) = (2x + y + z, x + 2y + z)$ relative to the ordered basis $B = \{(1, 1, 0), (0, 1, 1), (1, 0, 1)\}$ and $B^{/} = \{(1, 2), (2, 1)\}$.