# U.G. 3rd Semester Examination - 2020

# PHYSICS

## [HONOURS]

### Course Code : PHY-H-CC-T-05

### (Mathematical Physics-II)

Full Marks : 40 Time : $2\frac{1}{2}$ Hours

*The figures in the right-hand margin indicate marks.*

*Candidates are required to give their answers in their own words as far as practicable.*

1. Answer any **five** questions: 2×5=10

   i) What are ordinary and singular points of differential equation?

   ii) State two properties of Bessel function.

   iii) What is propagation of errors?

   iv) Evaluate $\Gamma\left(-\frac{1}{2}\right)$

   v) Solve $xdx + ydy + 4y^3(x^2 + y^2)dy = 0$

   vi) What is Laguerre's differential equation? Write down its solution.



   vii) State the conditions under which a function can be expanded into a convergent series.

   viii) What do you mean by systematic error?

2. Answer any **two** questions: 5×2=10

   i) Find the Fourier series of the function *f(x)* given by,

   $$f(x) = 1 - \frac{2x}{\pi} \text{ for } 0 \le x \le \pi$$
   $$1 + \frac{2x}{\pi} \text{for} -\pi \le x \le 0$$ 5

   ii) Solve the Differential equation by Frobenius method

   $$x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2 - n^{2)}y = 0$$ 5

   iii) State and Prove Rodrigue's Formula for Legendre polynomials. 5

   iv) Evaluate $\int_0^1 x^4(1-\sqrt{x})^5 dx$ 5

3. Answer any **two** questions: 10×2=20

   i) A tightly stretched string with fixed end points *x=0* and *x=l* is initially at rest in equilibrium position. If it is vibrating by giving to each of its point a velocity $\lambda x(l\text{-}x)$, find the displacement of the string at any distance *x* from one end at any time *t*. 10

ii) Solve the partial differential equation

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} - 2\frac{\partial^2 z}{\partial y^2} = e^{2x+y}$$ 10

iii) Starting from the generating function for Legendre polynomials prove the following recurrence relations:

a) $(l+1)P_{l+1}(x) - (2l+1)xP_l(x) + lP_{l-1}(x) = 0$

b) $lP_l(x) = xP_l'(x) - P_{l-1}'(x)$

Evaluate $\int_0^\infty e^{-x^2} \frac{d^2 H_m}{dx^2} H_m(x) dx$ 3+3+4

iv) a) Show that the equation,

$$x^2\frac{d^2 y}{dx^2} + 2x\frac{dy}{dx} + [x^2 - n(n+1)]y = 0,$$

where $n$ is a positive integer, can be transferred to Bessel equation of order $(n+\frac{1}{2})$ by substitution $y(x) = \frac{z(x)}{x^{1/2}}$.

b) Prove that $J_{n-1}(x) - J_{n+1}(x) = 2J_n'(x)$

c) Solve Laplace equation in three dimensional cylindrical form. 3+3+4

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