Kandi Raj College – Department of Mathematics – Internal Examination – 1st Semester – Honours course

Full Marks: CC – T – 01 = 10, CC – T – 02 = 10 [For students of Mathematics Honours]

HGE – T – 1 = 10 [For students opting for Mathematics as GE]

FOR MATHEMATICS HONOURS STUDENTS (MTMH) [USE SEPARATE ANSWER-SCRIPTS FOR CC-T-01 & CC-T-02]

CC-T-01 — 10

Group – A [Matrices & Linear Algebra] — **05**

Answer any ONE question:

1. If $y = (\sin^{-1} x)^2$, prove that, $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - n^2y_n = 0$ (Where the symbols have their usual meaning).

2. If $I_n = \int_0^1 x^n \tan^{-1} x\, dx$,($n > 2$, is a positive integer), then prove that, $(n + 1)I_n + (n - 1)I_{n-2} = \frac{\pi}{2} - \frac{1}{n}$.

Group – B [Geometry] — **02**

Answer any ONE question:

3. What does the equation $3x^2 - 4xy + 25y^2 = 0$ become when the axes turned through an angle $\tan^{-1} 2$.

4. Find the radius of the circle $x^2 + y^2 + z^2 = 49, 2x - y + 3z = 14$.

Group – C [Differential Equations] — **03**

Answer any ONE question:

5. Find the differential equation of all parabolas having origin as vertex and focus on y-axis.

6. Solve, $\frac{dy}{dx} + \frac{y}{x} = y^2$.

CC-T-02 — 10

Group – A [Algebra – units 1 & 2] — **05**

Answer any ONE question:

1. If the equations $x^2 + 2x + 3 = 0$ and $ax^2 + bx + c = 0, a, b, c \in \mathbb{R}$ have a common root then find $a: b: c$. — **2+3**

If a, b, c are the sides of a triangle, show that $\frac{1}{2} < \frac{ab+bc+ca}{a^2+b^2+c\text{^}2} < 1$.

2. Use strong principle of mathematical induction to show that, $\left(3 + \sqrt{7}\right)^n + \left(3 - \sqrt{7}\right)^n$ is an even integer for all $n \in \mathbb{N}$. — **3+2**

Show that the product of all values $\left(1 + \sqrt{3}i\right)^{\frac{3}{4}}$ is 8.

Group – B [Algebra – units 3 & 4] — **05**

Answer any ONE question:

3. Determine the conditions for which the following system of equations has (a) only one solution; (b) no solution; (c) infinitely many solutions.

$$\begin{gathered} x + y + z = 1 \\ x + 2y - z = b \\ 5x + 7y + az = b^2 \end{gathered}$$

4. (i) State Cayley – Hamilton theorem. — **1**

(ii) Using Cayley – Hamilton theorem for the matrix A, find A^{-1}, where, — **4**

$$A = \begin{bmatrix} 2 & -2 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

END OF QUESTIONS FOR MATHEMATICS HONOURS

FOR HONOURS STUDENTS OTHER THAN MATHEMATICS (PHSH , CEMH)

HGE – T – 01 **10**

Answer any TWO questions: **2 × 5**

1. A function $f: \mathbb{R} \to \mathbb{R}$ is defined by, $f(x) = \begin{cases} x, & x < 1 \\ 2 - x, & 1 \le x \le 2 \\ x^2 - 3x + 2, & x > 2 \end{cases}$

Show that $f'(x)$ does not exist at 1 and 2.

2. If $u = \tan^{-1}\left\{\frac{x^3+y^3}{x-y}\right\}$, then show that, $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$.

3. Find the radius of curvature of $y = xe^{-x}$ at its maximum point.

END OF QUESTIONS FOR HONOURS GENERAL