Kandi Raj College – Department of Mathematics – Internal Examination – 6th Semester – Honours course

Full marks: CC-T-13 = 10 ; CC-T-14 = 10 ; DSE-T-03 = 10 ; DSE-T-04 = 10

	CC – T – 13	**10**
1.	Answer any 1(One) question.	$5 \times 1 = 05$
a.(i)	Show that the differentiability of a function $f(z)$ at a point z_0 implies the continuity of the function $f(z)$ at z_0.	03
(ii)	Show by an example that the converse of the above statement is not true.	02
b.(i)	State Cauchy-Gaursat theorem. Find $\int_{\lvert z\rvert=1} dz/(z-2)$.	02
(ii)	Find $\int_{\lvert z\rvert=2} \frac{(e^z+z^2)dz}{(z-1)}$.	03
2.	Answer any 1(One) question.	$5 \times 1 = 05$
a.	Show that continuous image of a compact metric space is compact.	05
b.	Let (X, d) and (Y, ρ) be two metric spaces. Let $f: X \to Y$ be a bijective mapping. Show that f is a homomorphism, if and only if $f(\bar{A}\ ) = \overline{f(A)}$ for any subset A of X.	05

	CC – T – 14	**10**
1.	Answer any 2(Two) questions.	$5 \times 2 = 10$
a.	If $B = \{(-1,1,1), (1,-1,1), (1,1,-1)\}$ is a basis of $V_3(R)$, then find the dual basis of B.	05
b.	Find the minimal polynomials of the matrix $\begin{pmatrix} 3 & 1 & -1 \\ 2 & 4 & -2 \\ -1 & -1 & 3 \end{pmatrix}$.	05
c.	Apply Gram Schmidt Process to the set $\{(1,1,1),\ (2,-2,1),\ (3,1,2)\}$ to obtain an orthonormal basis of $\mathbb{R}^3$ with standard inner product.	05

	DSE – T – 03	**10**
1.	Answer any 2(Two) questions.	$5 \times 2 = 10$
a.(i)	If $p \geq 5$ is a prime number, show that $p^2 + 2$ is composite.	02
(ii)	If p and $p^2 + 8$ are primes, then show that $p^3 + 4$ is also a prime.	03
b.	Prove that $\phi(3n) = 3\phi(n)\ iff\ 3\lvert n$.	05
c.	Let p be a prime of the form $4n \pm 1$. Prove that every positive factor d of n is a quadratic residue of p.	05

	DSE – T – 04	**10**
1.	Answer any 2(Two) questions.	$5 \times 2 = 10$
a.	A uniform sphere is held in equilibrium on a rough inclined plane of angle α, by a force of magnitude $\frac{1}{2}w \sin\alpha$, applied tangentially to its circumference, where w is the weight of the sphere. Prove that the force must act parallel to the plane and that the coefficient of friction must be $\frac{1}{2}\tan\alpha$.	05
b.	Two forces $2P$ & P act along the lines with equations $y = x\tan\alpha\,, z = c, y = -x\tan\alpha\,, z = -c$ respectively. Find the equation of the central axis.	05
c.	A heavy particle slides down a smooth cycloid, starting from rest at the cusp, axis vertical, vertex downwards. Prove that the magnitude of acceleration is equal to g at every point of the path and the pressure, when the particle arrives at the vertex, is equal to twice the weight of the particle.	05