2021
MATHEMATICS
[HONOURS]
Paper : V

Full Marks : 100 Time : 4 Hours

The figures in the right-hand margin indicate marks. Candidates are required to give their answers in their own words as far as practicable.

Symbols/Notations have their usual meanings.

1. Answer any **five** questions: 1×5=5

   a) Define length of simple equivalent pendulum.

   b) Define fluid pressure.

   c) What is metacentre?

   d) Define central axis.

   e) What is conservative field of force?

   f) Define coefficient of friction.

   g) State Lami's theorem.

   h) What do you mean by two-dimensional motion of a rigid body?

[Turn over]

2. Answer any **ten** questions: 2×10=20

   a) Prove that for a reversible adiabatic change $PV^{\gamma} =$ constant. (Notations are usual)

   b) Show that for a reversible isothermal change of one mole gas PV=constant.

   c) Write the equations of motion of a rigid body moving in two dimension under impulsive forces.

   d) Calculate the kinetic energy of a rigid body moving about a fixed axis.

   e) Explain D'Alembert's principle.

   f) State the necessary and sufficient conditions that two systems to be equimomental.

   g) Find the distance of C.G. of a sector of a circle from its centre.

   h) Three forces P, Q, R act along the lines OB, OA, AB, where A and B are the points of interaction of the straight lines with the axes OX and OY respectively. Find the magnitude of their resultant.

   i) State the principle of virtual work.

   j) A uniform beam of thickness 2b rests symmetrically on a perfectly rough horizontal cylinder of radius a. Show that the equilibrium of the beam will be stable if $b < a$.

k) Prove that in a homogeneous fluid rest under gravity, the pressure difference between two points is proportional to the difference of their depths.

l) Prove that the pressure at depth h below the free surface of a homogeneous liquid of density ρ is ρgh.

3. Answer any **five** questions: $6\times5=30$

a) A uniform rod of length 2a, is placed on a rough table at right to its edge. If its centre of gravity be initially at distance 'b' beyond the edge, show that the rod will begin to slide when it has turned through an angle $\tan^{-1}\left(\frac{\mu a^2}{a^2+9b^2}\right)$, where μ is coefficient of friction.

b) A given volume V of liquid is acted upon by forces $-\frac{\mu x}{a^2}, -\frac{\mu y}{b^2}, -\frac{\mu z}{c^2}$, per unit mass along the x, y, z direction respectively. Find the equation of the free surface.

c) A quadrant of a circle is just immersed vertically in a liquid with one edge in the free

surface. Find the position of the centre of pressure if the density of the liquid varies as the depth.

d) A conical vessel of height h and vertical angle 2α contains water whose volume is one-half of the cone. If the vessel and the contained water revolve with uniform angular velocity ω, and no water overflows, show that ω must not be greater than $\sqrt{\frac{2g}{3h}}\cot\alpha$.

e) Investigate the condition of equilibrium of a particle constrained to rest on a rough plane curve $f(x, y)=0$ under any given forces in the plane of the curve.

f) The forces X, Y, Z act along the straight lines y=0, z=1; z=0, x=1; x=0, y=1. Prove that the pitch of the equivalent wrench is

$$\frac{YZ+ZX+XY}{X^2+Y^2+Z^2}.$$

g) A heavy uniform rod of length 2a and mass m can turn freely about one end which is fixed. If it starts with angular velocity ω from the position in which it hangs vertically, find the angular velocity. Also prove that the time of describing an angle θ is $2\sqrt{\frac{a}{3g}}\log\tan\left(\frac{\pi}{4}+\frac{\theta}{2}\right)$.

h) Define angular momentum of a rigid body about a line. State and prove the principle of conservation of angular momentum of a rigid body moving under finite and impulsive forces.

Answer any **three** questions: 15×3=45

4. a) Define principal axis. Obtain the conditions for a given straight line to be a principal axis of the system of some point of its length. If so, find the other two principal axes.

b) A non-homogeneous rod AB of length $2l$, whose density at any point is directly proportional to the distance of the point from A, is rotating with a uniform angular velocity ω about a vertical axis through A. If the rod incliend at an angle α to the vertical, show that the value of α is either 0 or $\cos^{-1}\left(\frac{2g}{3l\omega^2}\right)$.

7+8

5. a) When the depth of the fluid is increased by an amount a, the depth of centre of pressure is found to be increased by y, and when instead, the depth of the liquid is increased by b, that of the centre of pressure is found to increase by z. Show that the centre of gravity in the original state of the liquid is $\frac{ab(b-a+y-z)}{az-by}$.

b) Define metacentre of a floating body. Prove that for a body floating freely in a heavy homogeneous liquid at rest, the metacentre exists if the axis of rotation is a principal axis of rotation of the plane of rotation at the same point. Hence show that $HM=\frac{AK^2}{V}$, where the symbols have their usual meanings. 7+8

6. a) A cone of density σ, height h, radius of the base a and semi-vertical angle α floats in a liquid of density ρ with its axis vertical and vertex upwards. Show that equilibrium will be stable if $\frac{\sigma}{\rho}<1-\cos^6\alpha$.

b) A fluid is at rest under a given system of external forces. Find the pressure equation. If the external field of force be conservative, show that the surfaces of equipressure, equidensity and equipotential energy coincide.

7+8

7. a) A force parallel to z-axis acts at the point $(a, 0, 0)$ and an equal force perpendicular to z-axis acts at the point $(-1, 0, 0)$. Show that the central axis of the system lies on the surface $z^2(x^2+y^2)=(x^2+y^2-ax)^2$.

b) A quadrilateral ABCD formed of four uniform rods freely jointed to each other at their ends, the rods AB, AD being equal and also the rods BC, CD are freely suspended from the point A. A string joins A to C and such that $\angle ABC$ is a right angle. Prove by using principle of virtual work that the tension in the string is $(\omega+\omega')\sin^2\theta+\omega'$, where ω is the weight of the upper rod and ω' of a lower rod and $2\theta=\angle BAD$. 7+8

8. a) A body rests in equilibrium on another fixed body rough enough to prevent sliding. If the portion of two bodies in contact are spherical of radii r and R respectively and in equilibrium position the line joining their centres is vertical then show that the equilibrium is stable if $\frac{1}{h}>\frac{1}{r}+\frac{1}{R}$ where h is the height of C.G. of the body in position of equilibrium above the point of contact.

b) A homogeneous sphere of radius 'a' is rotating with an angular velocity ω about a horizontal diameter. It is then gently placed on a table whose coefficient of friction is μ. Show that there will be slipping at the point of contact for a time $\frac{2\omega a}{7\mu g}$ and then the sphere will roll with angular velocity $\frac{2\omega}{7}$. 8+7
