2021
PHYSICS
[HONOURS]
Paper : VIII

Full Marks : 80 Time : 4 Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP-A

1. Answer any **seven** questions: 1×7=7

a) Show that the phase-space trajectory of a one-dimensional simple harmonic oscillator is an ellipse.

b) Find the dimension of Planck's constant.

c) Write down the expression of de Broglie wavelength of a particle with kinetic energy T.

d) What is 'Bohr Magneton'? Give its value.

e) Give an expression of the wave function in the momentum space which satisfies Scroedinger wave equation.

f) What are configuration space and phase space variables?

g) Show that the equations of motion remain unchanged, if a total derivative term appears in the Lagrangian.

h) Mention the significance of Liouville's theorem.

i) State and explain equal a priori probability.

GROUP-B

2. Answer any **six** questions: 2×6 =12

a) Establish Rayleigh-Jeans law from Planck's radiation formula in the large wavelength limit.

b) Given the radius of the first Bohr orbit of Hydrogen atom to be $0.527Å$, show that a full de Broglie wave of an electron may be accommodated in it.

c) Derive an expression of Planck's constant in terms of stopping potentials of a metal, V_{s1} and V_{s2} , corresponding to two different wavelengths λ_1 and λ_2 respectively.

d) Calculate the permitted energy levels of an electron in a one-dimensional box of width $1Å$ ·

e) Explain the two major problems associated with constraints which compelled to modify

Newton's law.

f) Prove D'Alembert's principle.

g) Which value of f(p) makes the transformations

$Q = \frac{f(p)}{m\omega}\sin q;\ P = f(p)\cos q$ canonical?

h) Prove Boltzmann relation connecting the entropy and the number of microstates.

GROUP-C

3. Answer any **three** questions: 7×3=21

a) Derive probability distribution function for a closed system. Find the expressions for entropy of an ideal gas in the closed system, and explain Gibbs' paradox. 2+3+2

b) i) Find out the commutation relation between two operators to have simultaneous eigenfunctions.

ii) Prove that the probability current density for a three dimensional wave function $\Psi = e^{ikr}/r$ is given by $J = v/r^2$, where 'v' is velocity of the particle.

iii) Show that two stationary state solutions for the time independent Schrodinger equation corresponding to different

energy eigenvalues E_1 and E_2 are orthogonal. 2+2+3

c) i) Find the commutator $\left[\frac{\partial^2}{\partial x^2}, x\right]$, and hence show that the result may be expressed in terms of the momentum operator as $\left[\hat{p}^2, \hat{x}\right] = -2i\hbar\hat{p}$.

ii) Establish Heisenberg's uncertainty relation between position and momentum in case of a quantum one-dimensional harmonic oscillator, starting from the expression of discrete energy eigenvalues. 3+4

d) Derive the equation of motion of a simple pendulum both in view of D'Alembert's principle and using Lagrange multiplier technique. Explain the need for generalized coordinates. 2+3+2

e) Write down the Lagrangian and find the equation of motion of a particle of mass m moving with a velocity v near the surface of earth's rotating co-ordinate system. Briefly explain each term. Also find the expression for the Hamiltonian of the system. 3+2+2

GROUP-D

4. Answer any **four** questions: 10×4 = 40

a) i) Show that Maxwell-Boltzmann energy distribution law is a limiting case of Bose-Einstein and Fermi-Dirac statistics.

ii) What is meant by electron gas? Derive the expression for energy distribution of free electrons in metals.

iii) Copper contains 8.5×10^{34} free electrons per cc. Assuming that copper atoms donate one free electron, calculate the Fermi energy. Also find the number of quantum states in the energy range $\in_F \pm kT$ for the free electrons at 300 K in 1cc. of gas. 3+4+3

b) i) What is anomalous Zeeman effect? Calculate Lande g-factors for different levels and indicate the possible transitions between magnetically split Sodium Dl and D2 lines.

ii) Determine the probability current density for the wave function given by $\psi = A\exp\left(-\frac{\alpha^2x^2}{2}\right)\exp(ikx)$ evaluating

the normalization constant.

(1+4) + (1+4)

c) i) State Ehrenfest's theorem. Establish it in case of dynamical variables relating position and momentum of a quantum mechanical particle.

ii) A potential step is defined by V=0 for the region x<0 and $V = V_0$ for x>0. Find the 'reflectance' and 'transmittance' at the potential discontinuity. Show that the incident wave in the negative x-region is totally reflected if the particle has energy 'E' < V_0. (1+3)+(2+2+2)

d) Write down the Lagrangian and compute the equation of motion of a charged particle in an electromagnetic field. Find the Hamiltonian and the Hamiltonian equations of motion.

2+4+4

e) The bobs of two pendulums having equal length and mass are attached by a spring and set in motion. Find the Euler-Lagrange equations of motion of the system. Under the assumption of small oscillation, find the secular determinant and the normal modes of vibration. Also find the general solutions, using orthonormal condition. 3+2+2+3

f) Estimate the statistical count to find the expression for Bose-Einstein distribution function, and hence derive Planck's law of radiation. Discuss Debye's theory for the specific heat of solids. 4+2+4

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