U.G. 6th Semester Examination - 2021

MATHEMATICS

[HONOURS]

Discipline Specific Elective (DSE)

Course Code : MATH-H-DSE-T-04A

(Mechanics)

Full Marks : 60 Time : $2\frac{1}{2}$ Hours

The figures in the right-hand margin indicate marks.

The symbols and notations have their usual meanings.

1. Answer any **ten** questions: 2×10=20

a) Prove that at an apse, the planet moves at right angle to the radius vector.

b) Define catenary of uniform strength. Write down its equation in rectangular Cartesian coordinates.

c) Explain the energy test of stability for a body resting on a fixed body.

d) Show that the sum of the moments of inertia of a body about any three mutually perpendicular axes is constant.

e) State D' Alembert's principle.

f) Deduce the relation between linear velocity and angular velocity of a particle moving on a smooth plane curve.

g) State the principle of virtual work for a system of forces in equilibrium.

h) Find the product of inertia of an elliptic lamina with respect to its major and minor axes.

i) A particle describes a curve $r = ae^{\theta}$ with constant angular velocity. Show that its transverse acceleration varies as the distance from the pole.

j) Define angle of friction and cone of friction.

k) State the principle of conservation of linear momentum for a system of conservative forces.

l) What do you mean by constraint on a dynamical system? Give an example of it.

m) Prove that a central orbit is a plane curve.

n) State Kepler's laws of planetary motion.

o) Define a parking orbit.

2. Answer any **four** questions: 5×4=20

a) Two uniform rods, each of W weight and length a are freely joined at A and each passes over a smooth peg at the same level. From A, a weight W' is suspended. Show that in the position of equilibrium the inclination θ of the rod to the horizon is given by

$$\cos^3\theta = \frac{c(2W+W')}{2aW}.$$

b) Show that the kinetic energy of a rigid body, moving in any manner is at any instant equal to the kinetic energy of the whole mass, supposed to be collected at its centre of inertia and moving with it together with the kinetic energy of the whole mass relative to its centre of inertia.

c) A particle describes a path which is nearly a circle about a centre of force $\mu\phi(u)$ per unit mass, u being the reciprocal of the distance from the centre of force. Find the condition that this may be a stable motion.

d) A particle of unit mass is projected with velocity u at an inclination α above the horizon in a medium whose resistance is k times its velocity. Show that its direction will again make an angle α with the horizon after a time $\frac{1}{k}\left\{1+\frac{2ku}{g}\sin\alpha\right\}$.

e) If a planet is suddenly stopped in its orbit supposed circular, show that it would fall into the sun in a time which is $\frac{\sqrt{2}}{8}$ times the period of the planet's revolution.

f) A uniform rod OA of length $2a$ free to turn about its end O, revolves with angular velocity ω about the vertical OZ through O, and is inclined at a constant angle α to OZ, show that the value of α is either zero or $\cos^{-1}\left(\frac{3g}{4a\omega^2}\right)$.

3. Answer any **two** questions: 10×2=20

a) i) What do you mean by principal axes? Find the condition that a given straight line may be a principal axis at any point on the line and if the line is a principal axis then determine the other two principal axes.

ii) Show that the momental ellipsoid at the centre of an elliptic plate is

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+z^2\left[\frac{1}{a^2}+\frac{1}{b^2}\right]=\text{constant.}$$

(1+5)+4

b) i) Show that the kinetic energy of a rigid body moving in two dimensions is given by $\frac{1}{2}Mv^2+\frac{1}{2}Mk^2\dot{\theta}^2$.

ii) A uniform rod of mass M is placed at right angles to a smooth plane of inclination α with one end in contact with it. The rod is then released. Show that when the inclination to the plane is ϕ, the reaction of the plane will be

$$Mg\left[\frac{3(1-\sin\phi)^2+1}{(1+3\cos^2\phi)^2}\right]\cos\alpha.$$ 5+5

c) What is Poinsot's central axis? Derive its equations. Two forces $2P$ and P act along the lines whose equations are $y=x\tan\alpha$, $z=c$ and $y=-x\tan\alpha$, $z=-c$ respectively. Find the equation of the central axis. 2+3+5

d) i) Suppose a particle is moving in a plane. Besides the central acceleration F it has an acceleration T in the direction perpendicular to F. Show that the differential equation of the path is

$$\frac{d^2u}{d\theta^2}+u=\frac{F-\frac{Tdu}{ud\theta}}{h^2u^2}.$$

ii) Find the law of force to the pole when the path is the cardioid $r=a(1-\cos\theta)$ and prove that if F be the force at the apse and v be the velocity there, then $3v^2=4aF$. 5+5

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