U.G. 6th Semester Examination - 2021

MATHEMATICS

[HONOURS]

Discipline Specific Elective (DSE)

Course Code : MATH-H-DSE-T-04B

(Biomathematics)

Full Marks : 60 Time : $2\frac{1}{2}$ Hours

The figures in the right-hand margin indicate marks.

The symbols and notations have their usual meanings.

1. Answer any **ten** questions: 2×10=20

   a) Define autonomous system of a dynamical system.

   b) What is hyperbolic equilibrium point of a system?

   c) State Hartman-Grobman theorem.

   d) What do you mean by 'bifurcation' of a dynamical system?

   e) State Routh-Hurwitz criterion.

   f) What is epidemic model?

[Turn Over]

   g) State the basic equations of classical Lotka-Volterra model for a prey-predator system.

   h) What is "Diffusive instability"?

   i) State Bendixson's negative criterion.

   j) Define asymptotic stability of the system around an equilibrium point.

   k) What is allee effect in biological systems?

   l) Define Basic reproduction number in epidemics.

   m) What is Michaelis-Menten function in chemical kinetics?

   n) Write the differential equations of growth of a microbial population on a single resource in a chemostat.

   o) What is activator-inhibitor system?

2. Answer any **four** questions: 5×4=20

   a) Consider the system: $\frac{dx}{dt} = ax - 2y$, $\frac{dy}{dt} = -2x$, check the stability of the system for different values of $a \in R$. 5

   b) Check whether the Hartman-Grobman theorem is applicable to the system

$$\frac{dx}{dt} = ax - bx^2 - cxy, \quad \frac{dy}{dt} = cxy - dy$$

of equilibrium point (0,0), $a, b, c, d > 0$. 5

c) Describe Nicholson-Bailey model with all state variables and parameters. Find the equilibrium point of the model. 3+2

d) Find the phase path and draw it on the phase diagram

$$\frac{dx}{dt} = x + y, \quad \frac{dy}{dt} = x - y. \qquad 5$$

e) Deduce Hardy-Weinberg frequencies in genetics. 5

f) What is Kermack-McKendrick model in epidemics? Find the Basic reproduction number of the system. Hence find the stability of the endemic equilibrium. 2+2+1

3. Answer any **two** questions: 10×2=20

a) Propose a SIR epidemic model with birth and death. Find all biologically feasible equilibrium points of the proposed model. Find the stability conditions of the endemic equilibrium. 3+4+3

b) Consider the system: $\frac{dx}{dt} = rx\left(1 - \frac{x}{k}\right) - h$. Find the equilibrium points and check the stability of the system around the equilibrium points. Also find the critical value of h for a bifurcation to occur (where r, k, h are all positive parameters). 3+4+3

c) Consider a simple model for a population with an exponential growth and simple Fickian equation $\frac{\partial n}{\partial t} = rn + D\frac{\partial^2 n}{\partial x^2}$, $n(0, t) = n(L, t) = 0$; $n(x, 0) = n_0(x)$. Find the population size at any time t. Also find the critical length for which the population grow. (where r and D are positive parameters). 8+2

d) Consider the system $\frac{dx}{dt} = rx\left(1 - \frac{x}{k}\right) - qxy$, $\frac{dy}{dt} = qxy - ay$, find the interior equilibrium point and check the stability of the system around that equilibrium point, where r, k, q, a are all positive parameters. 5+5

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