U.G. 6th Semester Examination - 2021

STATISTICS

[PROGRAMME]

Discipline Specific Elective (DSE)

Course Code : STAT-G-DSE-T-1B&1D

Full Marks : 50(40+10) Time : $2\frac{1}{2}$ Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

Notations and symbols have their usual meanings.

Answer all the questions from selected Option.

OPTION-A

STAT-G-DSE-T-1B

(Survival Analysis)

1. Answer any **five** questions: 2×5=10

   a) Distinguish between truncation and censoring.

   b) What do you mean by duration of an epidemic?

   c) How would you derive the hazard function when you have cumulative hazard function?

   d) Define IFR. Cite an example where we can observe this.

   e) Define mean residual life.

   f) What is median survival time?

   g) Write a short note on Type II censoring.

   h) Why is random censoring more relevant in the context of survival analysis?

2. Answer any **two** questions: 5×2=10

   a) Find out the hazard function for a time to event variable that follows Gamma distribution. Discuss on the shape of the hazard functions with different parameter choices.

   b) Write a note on minimum chi-square methods in the context of competing risk theory.

   c) Discuss New Better than Used (NBU), New Worse than Used (NBU) and Loss of Memory (LOM) properties.

   d) Derive the expression for the mean residual life for a time to event variable following Gamma distribution.

3. Answer any **two** questions: 10×2=20

   a) i) Derive Kaplan Meier estimator for a group of tied censored failure time data. 7

   ii) Discuss how would you obtain the estimator of cumulative hazard function using Kaplan Meier form. 3

b) i) Obtain the variance of the estimated survival function, derived using Kaplan Meier method. 8

ii) Discuss on the difference between Kaplan Meier method and Actuarial method. 2

c) i) Discuss about the bathtub shaped failure rate life distributions. 5

ii) Suppose the time to event (T) random variable has a Weibull distribution with the following cdf:

$F_{\alpha}(t) = 1 - e^{-(\lambda t)\alpha}$, for $t \geq 0$, where, $\alpha, \lambda > 0$.

Show that it is IFR for $\alpha \geq 1$ and DFR for $0 < \alpha \leq 1$. 5

d) i) Assuming time (t) to be continuous, discuss the relationship between hazard function and survival function. 5

ii) Show that the hazard function for an exponentially distributed time to event variable is constant. 2

iii) What do you mean by dependent risk model? 3

[*Internal Assessment: 10*]

OPTION-B

STAT-G-DSE-T-1D

(Actuarial Statistics)

1. Answer any **five** questions: $2\times5=10$

a) If the force of interest per annum is 0.06 and if Rs.40,000 is invested on 1st January 2020, obtain the accumulated value on 1st January 2021.

b) Explain the term 'insurable risk'. State its characteristics.

c) Explain the following concepts:

i) Policy

ii) Rate of discount

d) Obtain accumulated value of Rs.40,000 at the end of 7th year with effective rate of interest 6%.

e) Define utility function with an example.

f) Find the present value of annuity certain immediate at the rate of 1 unit per annum for n years.

g) A person has invested Rs.1,000 in a savings scheme. After 15 years he is entitled to receive Rs.1,750. What rate of interest is realized in the transaction?

h) Express e_x in terms of l_x.

2. Answer any **two** questions: 5×2=10

a) A loan of Rs. 50,000 is taken and it has to be repaid in five equal installments payable yearly at the beginning of the year. Based on 6% annual effective rate of interest determine the amount of installments.

b) Show that the condition for mutually advantageous policy is $G \geq \mu$, where G is onetime premium and μ is expected value of loss random variable. State the assumptions.

c) The survival rates p_x for a certain population are as follows:

Age (yrs) x	0	1	2	3	4	5
p_x	0.85	0.8	0.4	0.5	0.2	0

i) Construct the columns l_x, L_x, T_x for a cohort of 100000.

ii) Obtain the limiting age w.

d) Define curate future lifetime random variable K(x) and find its probability mass function.

3. Answer any **two** questions: 10×2=20

a) i) State any two properties of survival function S(x). Derive the expression for S(x) in terms of constant force of mortality. 5

ii) Examine whether the following functions can serve as survival functions for $x \geq 0$.

A) $\left[S(x) = \exp\left(x - 0.7\left(2^x - 1\right)\right)\right)$

B) $S(x) = \frac{1}{(1+x)^2}$ 5

b) i) Explain the term ‘Annuity’ with illustration. 5

ii) On 5th June 2000, (65) bought a Rs. 1,00,000 whole life insurance policy with death benefit payable at the end of the year of death. The policy is purchased by means of annual premiums payable at the start of each year policy remain in force. The policy holder died on 10th August 2007 and the loss to the insurer was Rs. 30,000. If i=0.6, what was the annual premium paid? 5

c) i) Explain with an illustration each of the following:

A) N-year term insurance

B) Whole life insurance 4

ii) If for the annuity certain the payments are made regularly at the beginning of the year then derive that

$$\ddot{S}_n = (1+i)^n \ddot{a}_n$$ 6

d) It is known that

q_{28}=0.135, q_{29}=0.146, q_{30}=0.159,

q_{31}=0.173, q_{32}=0.188, and i=0.05

Obtain the annual premium paid at a 5 year temporary discrete annuity due, for a benefit of 1000, payable at the end of the year of death, in a 5 year term insurance, issued to (28). 10

[*Internal Assessment: 10*]

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