

**2021**  
**MATHEMATICS**  
**[HONOURS]**  
**Paper : VIII**

Full Marks : 50

Time : 2 Hours

*The figures in the right-hand margin indicate marks.**Symbols and Notations have their usual meaning.*

**GROUP-A**  
**(Numerical Analysis)**  
**(Marks : 30)**

1. Answer any **two** questions: 1×2=2
- a) What is the order of the error of Simpson's  $\frac{1}{3}$ rd composite formula for numerical evaluation of an integral  $\int_a^b f(x)dx$ .
- b) Find the absolute error, relative error and percentage error in the approximate value  $x_A=0.333$  when the true value  $x_T = \frac{1}{3}$ .
- c) Define polynomial interpolation. Comment on its uniqueness.

- d) What is the number of multiplications involved in solving a system of n linear equations with n unknowns by Gauss-Elimination method?
2. Answer any **three** questions: 2×3=6
- a) Solve the equation  $\frac{dy}{dx} = xy + 1$  when  $y(0)=1$  by Picard's method, upto  $x^3$  as the highest degree term.
- b) Indicate the situations where the fixed-point iteration process may not converge to a solution of the equation  $f(x) = 0$ .
- c) What is meant by divided difference  $f(x_0, x_0, x_0)$  for coincident values of the argument?
- d) Evaluate  $\Delta^{10} (1 - px)(1 - qx^2)(1 - rx^3)(1 - sx^4)$ .
- e) What is pivoting? State the difference between partial pivoting and complete pivoting.
3. Answer any **two** questions: 6×2=12
- a) Derive Newton-Raphson's method for finding real simple root of  $f(x)=0$  and discuss its convergence.
- b) Write down the Lagrange's interpolation formula and therefore deduce closed type Newton-Cote's formula in the form

$$I = (b-a) \sum_{r=0}^n K_r^{(n)} y_r \quad \text{for the integral}$$

$I = \int_a^b f(x) dx$ ,  $K_r^{(n)}$  being Cote's coefficients and  $y_r = f(x_r)$ .

- c) Explain the Gauss-Seidel iterative method of solving a system of linear equations. Give sufficient conditions for convergence of the process.
- d) Define the degree of precision of a numerical integration formula and hence determine the degree of precision of Simpson's  $\frac{1}{3}$ rd rule over  $[0, 2]$  with proper choice of the test functions. Does the formula having larger degree of precision yield better accuracy always?

4. Answer any **one** question: 10×1=10

- a) i) Derive the second order Runge-Kutta method for solving the differential equation  $\frac{dy}{dx} = f(x, y)$  with  $y(x_0) = y_0$  for  $x \in [x_0, x_n]$  and find the error involved in it. 5
- ii) Solve  $\frac{dy}{dx} = 1 - y$ ,  $y(0) = 0$  in the range

$0 \leq x \leq 0.2$  by using modified Euler's method by choosing  $h=0.1$ .

Compare the result with exact solution. 5

- b) i) Construct Lagrange's interpolation polynomial for the function  $y = \sin \pi x$ , choosing the points

$$x_0 = 0, x_1 = \frac{1}{6}, x_2 = \frac{1}{2}. \quad 4$$

- ii) Show that the maximum error in linear interpolation for  $f(x)$  on  $[x_1, x_2]$  is given

by  $\frac{1}{8}(x_1 - x_0)^2 M$ , where

$$M = \max |f''(\xi)|, x_0 \leq \xi \leq x_1.$$

Hence or otherwise, determine an appropriate step-size in the construction of a table of  $f(x) = (1+x)^6$  on  $[0, 1]$  so that the transaction error remains less than  $5 \times 10^{-5}$ . 3+3

**GROUP-B**  
**(Fundamentals of Computer Science and**  
**Computer Programming)**  
**(Marks : 20)**

5. Answer any **two** questions: 2×2=4
- a) What do you mean by constant? Describe the four basic types of constants used in C.
- b) Write the expression in equivalent C form:  
 $x = y \log_{10}(\cos m) + z \sqrt{\sin^{-1}(|p|)}$ .
- c) What are the differences between ROM and RAM?
- d) How a source program is converted into an object program? Explain with a diagram.
6. Answer any **one** question: 6×1=6
- a) Write a C program to generate first N terms of the Fibonacci sequence. 6
- b) Write two reasons of using binary numbers, instead of decimal numbers, in the computer memory. Perform subtraction of the following decimal numbers after converting them into binary numbers using binary arithmetic and find the decimal equivalence of the result:  
 $(94.5)_{10} - (43.75)_{10}$ . 1+5

7. Answer any **one** question: 10×1=10
- a) i) What is the purpose of 'scanf' function? How is it used within a C program? Compare it with the 'getchar' function. 3+2
- ii) Write a C program to find the sum of the series correct upto 5 significant digits:  

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$$
 (when x given in degree/radian). 5
- b) i) What is an array? How multidimensional array is different from one-dimensional array? 3
- ii) Write a C program to find the sum of squares of the diagonal elements of a square matrix of order n×n using the concept of array. 7
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