2021
STATISTICS
[HONOURS]
Paper : IV

Full Marks : 75 Time : 4 Hours

The figures in the right-hand margin indicate marks. Candidates are required to give their answers in their own words as far as practicable.

Answer all the questions.

1. Attempt any **five** of the following questions: $1\times5=5$

   i) Find a relation between Δ and E.

   ii) State Markov's inequality.

   iii) What do you mean by transcendental equation? Provide an example of it.

   iv) Write down Stirling's approximation formula for n!.

   v) If $\{a_n\}$ converges to a and $\{b_n\}$ converges to b then where does $\left\{\frac{a_n}{b_n}\right\}$ converge, if $b_n \neq 0$, $b \neq 0$?

[Turn over]

   vi) Give the logic for which you can assume the dispersion matrix Σ, of a random vector $\underset{\sim}{X}$, to be positive semi definite.

   vii) Find $\Delta^4 (x-3)(x+11)(x-7)$ when $\Delta x = 1$.

2. Attempt any **six** of the following questions: $2\times6=12$

   i) Define an improper integral. Give some examples.

   ii) Establish a relation between Δ and D.

   iii) Define radius of convergence for a real power series.

   iv) Why we generally use polynomial interpolation formula?

   v) What is the characterization of multivariate normal distribution?

   vi) Prove that a necessary and sufficient condition for the convergence of a monotonic sequence is that it is bounded.

   vii) Can a sequence converge to more than one limit?

   viii) Let (X, Y) be a bivariate random variables of continuous type. Define conditional distribution of Y given X=x. Hence define independence of X and Y.

ix) Let $\underset{\sim}{X} = (X_1, X_2)'$ have a bivariate normal distribution with $E(X_1) = E(X_2) = 0$, $E(X_1^2) = E(X_2^2) = 1$ and $E(X_1, X_2) = 1$. Find $P[|2x_1 + 3x_2| \leq 15]$.

3. Attempt any **three** of the following questions: 6×3=18

i) Given n positive numbers $C_1, C_2, ..., C_n$, find the maximum value of $\sum_{i=1}^{n} C_i x_i$; if the variables x_i's are so restricted so that $\sum_{i=1}^{n} x_i^2 = 1$.

ii) Define convergence in probability. Let $\{X_n\}$ be a sequence of random variables and X be another random variable defined on the same probability space, show that if $X_n \xrightarrow{P} X$ as $n \to \infty$ then $g(X_n) \xrightarrow{P} g(X)$ as $n \to \infty$ for any continuous function g(.).

iii) Distinguish between pointwise convergence and uniform convergence for a sequence of functions through an example.

iv) Suppose (X, Y) follows a Bivariate Normal distribution with parameters 0,0, 1, 1, ρ. If $q = P(XY > 0)$ then show that $\rho = \cos[(1-q)\pi]$.

v) Show that sum of the coefficients of entries in the Lagranges interpolation formula is unity.

4. Attempt any **four** of the following questions: 10×4=40

i) a) Show that the series $\sum_{n=1}^{\infty} \frac{\cos nx}{p}$ converges uniformly and absolutely if $p > 1$.

b) Show that the series $1 + x + x^2 + ... = \sum_{k=0}^{\infty} x^k$ converges to $S(x) = \frac{1}{1-x}$ on $[-a, a]$, $0 < a < 1$.

c) Show that the series $\sum_{n=1}^{\infty} (x e^{-x})^n$ of functions converges uniformly on $[0, 2]$. 2+5+3

ii) a) Show that the sequence $\left\{\frac{nx}{1+n^2x^2}\right\}$ of functions is not uniformly convergent on any interval containing zero.

b) Evaluate $\int_D xy\,dx\,dy$, where

$$D=\left\{(x,y)\in \mathbb{R}^2 \mid ax^2+by^2+2hxy\le r^2\right\}$$

with a, b, h, r $\in \mathbb{R}$ such that $r>0$, $a>0$ and $ab-h^2>0$. 4+6=10

iii) a) Let X and Y be random variables with means 0, variances 1 and correlation coefficient ρ. Show that

$$E\left[\max\left(X^2,Y^2\right)\right]\le 1+\sqrt{1-\rho^2}\,.$$

b) Using a) show that, for random variables X and Y with means μ_X and μ_Y, variances σ_X^2 and σ_Y^2, and correlation coefficient ρ

$$P\left[\left|X-\mu_x\right|\ge t\sigma_X \text{ or} \left|Y-\mu_y\right|\ge t\sigma_Y\right]\le \frac{1}{t^2}\left\{1+\sqrt{1-\rho^2}\right\}.$$

5+5=10

iv) State and prove weak law of large numbers by using Chebyshev's inequality. Also discuss the Bernoulli's theorem in this context.

5+5

v) a) Let X and Y be jointly distributed random variables such that they are uniformly distributed over the region $R=\{(x,\ y):0<x,y<1\}$. Find the conditional distribution of X given Y. Hence, find $\operatorname{Cov}(X,\ Y)$.

b) Let X and Y be jointly distributed random variables with joint p.d.f.

$$f(x,\ y)=\begin{cases}\frac{1+xy}{4} & \text{if} \quad |x|<1 \\ 0 & \text{otherwise}\end{cases}.$$

Show that X and Y are not independent but X^2 and Y^2 are independent.

5+5

vi) Explain the concept of partial correlation coefficient. Obtain a formula for $\rho_{12.34\ldots p}$. What is the interpretation of $\rho_{12.34\ldots p}=0$?

2+7+1
