2021
MATHEMATICS
[HONOURS]
Paper : IV

Full Marks : 100 Time : 4 Hours

The figures in the right-hand margin indicate marks.

Symbols have their usual meanings.

GROUP–A

(Linear Programming and Game Theory)

[Marks : 40]

1. Answer any **two** questions: 1×2=2

   a) Give an example of convex set in E^3.

   b) What do you mean by "Two person zero-sum game"?

   c) State fundamental theorem of LPP.

2. Answer any **two** questions: 2×2=4

   a) In which halfspace determined by the hyperplane $3x_1+2x_2+4x_3+6x_4=7$ does the point lie?

   b) If x_1, x_2 be real, show that the set given by $X=\left\{(x_1, x_2):9x_1^2+4x_2^2\le 36\right\}$ is a convex set.

   c) Show that whatever may be the value of a, the game with the following payoff matrix is strictly determinable:

B

		I	II
A	I	3	7
	II	−3	a

3. Answer any **four** questions: 6×4=24

   a) Find the minimum cost solution for the 4×4 assignment problem whose cost coefficients are as given below:

	I	II	III	IV
1	4	5	3	2
2	1	4	−2	3
3	4	2	1	−5

b) Solve the following L.P.P.:

Maximize $Z=60x_1+50x_2$

subject to $x_1+2x_2 \le 40$,

$$3x_1+2x_2 \le 60$$

$$x_1, x_2 \ge 0$$

c) Solve graphically or otherwise the game whose payoff matrix is given below:

	Player B		
Player A	3	−2	4
	−1	4	2
	2	2	6

d) Show that the feasible solution $x_1=1$, $x_2=1$, $x_3=0$ and $x_4=2$ to the system

$$x_1+x_2+x_3=2$$

$$x_1+x_2-3x_3=2$$

$$2x_1+4x_2+3x_3-x_4=4$$

$$x_1, x_2, x_3, x_4 \ge 0$$

is not basic.

e) Prove that a basic feasible solution to a Linear programming problem corresponds to an extreme point of the convex set of feasible solutions.

f) Solve the following transportation problem:

	D_1	D_2	D_3	D_4	a_i
O_1	10	7	3	6	3
O_2	1	6	8	3	5
O_3	7	4	5	3	7
b_j	3	2	6	4	

4. Answer any **one** question: 10×1=10

a) i) Prove that, if any of the constraints in the primal problem be a perfect equality, then the corresponding dual variable is unrestricted in sign.

ii) Use duality to solve the problem: 5+5=10

Maximize $Z=2x_1+3x_2$

subject to $-x_1+2x_2 \le 4$

$$x_1+x_2 \le 6$$

$$x_1+3x_2 \le 9$$

$$x_1, x_2 \ge 0$$

b) i) Show that, the number of basic variables in a transportation problem is at most (m+n–1).

ii) Solve the travelling salesman problem with the following cost matrix $[c_{ij}]_{4\times4}$ where c_{ij} is the cost of travelling from city i to the city j: 5+5

	1	2	3	4
1	∞	15	30	4
2	6	∞	4	1
3	10	15	∞	16
4	7	18	13	∞

GROUP–B

(Dynamics of a Particle)

[Marks : 50]

5. Answer any **two** questions: 1×2=2

a) State Kepler's third law of planetary motion.

b) Write down the Radial and Cross-radial components of velocity.

c) What is parking orbit?

6. Answer any **five** questions: 2×5=10

a) A particle describes a curve whose equation $\frac{a}{r}=\theta^2+b$ under a force to the pole. Find the law of force.

b) The displacement of a moving point at any point at time t is given by $x=a\cos kt+b\sin kt$. Show that the point executes a simple harmonic motion.

c) If the path of a particle be a circle, find its radial and cross-radial acceleration.

d) A particle thrown vertically upwards takes t secs. to rise to a height h and t' secs. is the subsequent time to reach the ground again. Show that $h=\frac{1}{2}gtt'$.

e) The velocity v of a particle moving in a straight line is given in terms of displacement S as $v^2=aS^2+2bS+c$, where a, b, c are constants. Show that the acceleration varies as the distance from a fixed point on the line.

f) If the angular velocity of a moving point about a fixed origin be constant, show that the transverse acceleration varies as its radial velocity.

g) A shell of mass 3 lbs is moving with a velocity 1200 ft/sec. when it bursts into two portions. One of them of mass 10 lbs moves on a velocity 5000 ft/sec. Find the velocity of the other piece.

7. Answer any **three** questions: 6×3=18

a) A particle describes a plane curve under an acceleration which is always directed towards a fixed point; find the differential equation of its path.

b) A particle is projected with velocity V from the cusp of a smooth inverted cycloid $r = a(1+\cos\theta)$ down the arc; show that the time of reaching the vertex is $2\sqrt{\frac{a}{g}}\tan^{-1}\left[\sqrt{\frac{4ag}{V}}\right]$.

c) A particle describes the equiangular spiral $r = ae^{m\theta}$ with a constant velocity. Find the components of the velocity and of the acceleration along the radius vector and perpendicular to it.

d) A particle moves towards a centre of force, the acceleration at a distance x being given by $\mu\left(x+\frac{a^4}{x^3}\right)$ where μ is a constant. If it starts from rest at a distance a, show that it will arrive at the centre in time $\frac{\pi}{4\sqrt{\mu}}$.

e) A heavy uniform chain of length $2l$, hangs over a small smooth fixed pulley, the length l+c being at one side and l–c at the other; if the end of the shorter portion be held and then let go, show that the chain will slip off the pulley in time $\left(\frac{l}{g}\right)^{\frac{1}{2}}\log\frac{l+\sqrt{l^2-c^2}}{c}\,(l>c)$.

8. Answer any **two** questions: 10×2=20

a) i) Find the intrinsic equation to a curve such that when a point moves on it with constant tangential acceleration, the magnitude of the tangential velocity and the normal acceleration are in constant ratio.

ii) A particle of mass m is attached to a light wire which is stretched lightly between two fixed points with a tension T. If a and b be the distances of the particle from the two ends, then prove that the period of a small transverse oscillation of the particle is $2\pi\sqrt{\dfrac{mab}{T(a+b)}}$. 6+4=10

b) i) A car of mass m from rest and moves on a level road under a constant frictional resistance, the engine working at a constant rate p. If the maximum speed is v and the speed u is attained after travelling a distance S in time t, show that $t=\dfrac{S}{v}+\dfrac{mu^2}{2p}$.

ii) If h be the height attained by a particle when projected, with a velocity V from the earth's surface supposing its attraction constant and H be the corresponding height when the variation of gravity is taken into account, prove that $\dfrac{1}{h}-\dfrac{1}{H}=\dfrac{1}{r}$, where r is the radius of the earth.

c) i) A particle is moving as a projectile under gravity. Show that the sum of the kinetic and potential energies at any point of its trajectory is constant.

ii) Prove that the kinetic energy of two particles of masses m and m' moving in a plane is $\dfrac{1}{2}(m+m')V^2+\dfrac{1}{2}\dfrac{mm'v^2}{m+m'}$, where V is the velocity of the centre of mass of the particles and v is the velocity of either of them relative to each other. 4+6=10

GROUP–C

(Analysis-II)

[Marks : 10]

9. Answer any **two** questions: 5×2=10

a) Find the maximum and minimum value of the function

$$\cos\cos\left(x-\frac{\pi}{6}\right)\cos\left(x+\frac{\pi}{6}\right)$$

where $0\leq x\leq\pi$.

b) A right circular cone with a flat circular base is constructed of sheet material of uniform small thickness. Express the total area of the surface in terms of volume and semi-vertical angle θ. Show that for a given volume, the area of the surface is minimum if $\theta = \sin^{-1}\left(\frac{1}{3}\right)$.

c) Evaluate:

i) $\lim_{x \to \infty}\left(\frac{1}{x^2} - \frac{1}{\sin^2 x}\right)$

ii) $\lim_{\theta \to 0} \frac{\theta \log \cos \theta}{e^{-\sin \theta} - 1 + \log(1+\theta)}$

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