6th-Sem-Internal Question for the Department of Mathematics (Kandi Raj College) – 2022

Question Paper for Honours students: Marks distribution: CC-13 = 10 ; CC-14 = 10 ; DSE-3 =10 ; DSE-4 = 10

[Answer all Papers in Separate answer sheets and upload separately:]

CC-13 [10]

[A] **Answer any One (1) question**

1. Let $f(z) = (|xy|)^{1/2}$. Show that $f'(0)$ does not exist but the C-R equations are satisfied at the origin. [5]

2. Evaluate the following integral by using Cauchy's integral formula [5]

$$\frac{1}{2\pi i}\int_C \frac{e^{zt}}{z^2+1}dz,\ t > 0$$

Where C is the circle $|z| = 3$.

[B] **Answer any One (1) question**

1. Prove that a metric space (X, d) is compact iff every family of closed subsets of X having FIP has nonempty intersection.

2. In C[0,1] with sup metric, show that $\{f_n\}$, where $f_n(x) = \frac{nx}{n+x},\ 0 \le x \le 1$ is a Cauchy sequence. Find $\lim_{n\to\propto} f_n$.

CC-14 [10]

Answer any Two (2) questions

1. Let a linear operator $T: R^2 \to R^2$ defined by $T(x,y) = (3x+3y, x+5y), (x,y) \in R^2$ prove that T is diagonalizable.
2. Let $B = \{(-1,1,1),(1,-1,1),(1,1,-1)\}$ be the basis of $V_3(R)$. Find the dual basis of B.
3. Show that set of all polynomials with even co-efficient is a prime ideal in $Z[x]$.

DSE-3 [10]

Answer any Two (2) questions

1. Prove that the linear system $x \equiv a\ (mod\ m)\ ;\ x \equiv b\ (mod\ n)$, is solvable if and only if $(m, n)|(a - b)$. [3+2]
When it is solvable, show that the solution is unique $modulo\ [m, n]$.

2. If p is an odd prime, show that $\left[\left(\frac{p-1}{2}\right)!\right]^2 + (-1)^{\frac{p-1}{2}} \equiv 0\ (mod\ p)$. [3+2]
Using it, show that, if $p \equiv 3\ (mod\ 4)$, then, $\left[\left(\frac{p-1}{2}\right)!\right]^2 \equiv 1\ (mod\ p)$.

3. Let $n = p_1^{k_1}p_2^{k_2}\cdots p_r^{k_r}$, for $n > 1$; if f is a multiplicative function, not everywhere zero, then show that, [4+1]

$$\sum_{d|n}\mu(d)f(d) = (1-f(p_1))\dots(1-f(p_r))$$

Use it to show that, $\sum_{d|n}\mu(d).d = (1-p_1)\dots(1-p_r)$

DSE-4 [10]

Answer any Two (2) questions

1. If each force of a system of coplanar forces be replaced by three forces, acting along the sides of a triangle in the plane of forces of type $p.BC, q.CA$ and $r.AB$, show that the necessary and sufficient conditions that the system reduces to a couple are that $\sum p = \sum q = \sum r$. [5]
2. Two heavy rings slide on a fixed smooth parabolic wire whose axis is horizontal and the rings are connected by a string which passes over a smooth peg at the focus. Prove that in the position of equilibrium the depths of the rings below the axis of the parabola are proportional to their weights. [5]
3. A heavy uniform rod of length $2a$, rests partly inside and partly outside a fixed smooth hemispherical bowl of radius r. The rim of the bowl is horizontal and one point of the rod is in contact with the rim. If θ be the inclination of the rod to the horizon, show that $2r\cos 2\theta = a\cos\theta$. [3+2]
Also show that the equilibrium of the rod is stable.