U.G. 6th Semester Examination - 2022

MATHEMATICS

[PROGRAMME]

Skill Enhancement Course (SEC)

Course Code : MATH-G-SEC-T-04(A)&(B)

Full Marks : 40 Time : 2 Hours

The figures in the right-hand margin indicate marks. Candidates are required to give their answers in their own words as far as practicable.

Answer all the questions from selected Option.

OPTION–A

MATH-G-SEC-T-04A

(Probability and Statistics)

1. Answer any **five** questions : 2×5=10

a) Find the probability of getting at least two heads in three throws of a coin.

b) Prove that $F(x)-F(x-0)=P(X=x)$, where F is a continuous probability distribution function of a random variable X.

c) Define probability density function for continuous random variable and prove the relation $F(x)=\int_{-\infty}^{x} f(t)\,dt$ where F and f denote probability distribution and probability density function respectively.

d) Find the mathematical expectation of the sum of points on m dice.

e) Find moment generating function of a random variable X whose pdf is

$$f(x)=\begin{cases}2x, & 0<x<1\\ 0, & \text{elsewhere}\end{cases}$$

f) Find the probability distribution function $F(x)$ of a random variable X whose probability density function is given by

$$f(x)=\begin{cases}3x^2, & 0<x<1\\ 0, & \text{elsewhere}\end{cases}$$

g) Let (X, Y) be two dimensional random variables and suppose that X and Y are independent. Then prove that $E(XY)=E(X)E(Y)$.

h) The probability, density function of a random variable is defined by

$$f(x)=\begin{cases}6x(1-x), & 0<x<1\\ 0, & \text{elsewhere}\end{cases}$$

Evaluate $P\left(X>\frac{2}{3}\right)$.

2. Answer any **two** questions: 5×2=10

a) A random variable X has the distribution given by $P(X=k)=2^{-k}, k=1,2,\ldots$ Show that $E(X)=\mathrm{Var}(X)=2$.

b) The bivariate random variable has the pdf

$$f(x,y)=\begin{cases}kx^2(8-y)\, x<y<2x,\ 0\le x\le 2\\ 0, \quad \text{elsewhere}\end{cases}.$$

Find k, the marginal probability density functions of X and Y and also the conditional pdf's $f_{\frac{x}{y}}\left(\frac{x}{y}\right)$, $f_{\frac{y}{x}}\left(\frac{y}{x}\right)$.

c) Find the moment generating function of a continuous probability distribution, whose density is $\frac{1}{2}x^2e^{-x}, 0<x<\infty$ and deduce the value of the mean and variance.

d) Let the probability density function of a random variable X is given by

$$f(x)=\begin{cases}c(4x-2x^2), & 0<x<2\\ 0, & \text{otherwise}\end{cases}$$

i) find the value of c,

ii) $P\left\{\frac{1}{2}<x<\frac{3}{2}\right\}=?$

3. Answer any **two** questions: 10×2=20

a) i) The joint probability density function of the random variable X and Y is given by

$$f(x,y)=\begin{cases}k(1-x-y)\, x\ge 0,\ y\ge 0,\ x+y\le 1\\ 0, \quad \text{otherwise}\end{cases},$$

where k is a constant. Find the marginal probability density functions, the mean value of y, when $x=\frac{1}{2}$, the covariance of X and Y. 5

ii) Find Mode and Median of the Binomial $(n,\ p)$ distribution. 5

b) i) Let X be a continuous random variable with probability density function f given by

$$f(x)=\begin{cases}ax, 0\le x\le 1\\ a,\ 1\le x\le 2\\ -ax+3a,\ 2\le x\le 3,\\ 0, \quad \text{elsewhere}\end{cases}$$

Determine the constant a. Determine the cumulative distribution function F and sketch its graph. 2+3+2

ii) If X has a gamma distribution with parameters $\alpha(>0)$ and $\lambda(>0)$ then show that the characteristic function $\theta(t)=\left(1-\frac{it}{\lambda}\right)^{-\alpha}$. 3

c) i) A bag contains 5 balls and it is not known how many of these are white. Two balls are drawn and are found to be white. What is the probability that all are white? 5

ii) If the moment generating function of a random variable W is $M(t)=(1-7t)^{-20}$, find the pdf, mean and variance of W. 2

iii) If the moment-generating function of X is $M(t)=\frac{e^{5t}-e^{4t}}{t}$, $t \neq 0$ and $M(0)=1$, find (A) E(X) (B)Var(x) and (C) $p(4.2<x\leq 4.7)$. 3

d) i) Determine the value of the constant k, such that the function $f(x,y)$, given by

$$f(x,y)=\begin{cases} k\dfrac{1+x+y}{(1+x)^4(1+y)^4}, & 0\leq x,y<\infty \\ 0, & \text{elsewhere} \end{cases}$$

is a probability density function of a bivariate distributiom (X, Y). Also find the marginal distribution of X and Y. 5

ii) If X is uniformly distributed over (0,12). calculate the probability that (A) $X<2$, (B) $X>8$, (C) $2<X<9$. 5

OPTION–B

MATH-G-SEC-T-04B

(Boolean Algebra)

1. Answer any **five** questions : 2×5=10

   a) Is $\mathbb{Z}$ a poset uuder the relation $a \preceq b$ if $a|b$?

   b) What do you mean by a chain? Give an example.

   c) Give an example of an ordered set P in which there are three elements *x, y, z* such that *{x, y, z}* is an antichain.

   d) Give an example of a poset which has exactly one maximal element but does not have a greatest element.

   e) Define latice homomorphism with an example.

   f) If L is a la tice and $a,b \in L$ then show that $a \vee (a \wedge b) = a$.

   g) What is a distributive lattice? Give an example.

   h) Give an example of a lattice which is not Boolean algebra.

2. Answer any **two** questions: 5×2=10

   a) Prove that any finite lattice is bounded. Find a lattice without a zero and a unit eletment. 3+2

   b) Show that the inverse of a lattice isomorphism is also a lattice isomorphism. 5

   c) Show that (*B, gcd, lcm*) is a Boolean algebra if *B* is the set of all positive divisors of 110. 5

3. Answer any **two** questions: 10×2=20

   a) i) Suppose that in a poset $b \vee c, a \vee (b \vee c)$ and $a \vee b$ exist. Prove that $(a \vee b) \vee c$ exists and that $a \vee (b \vee c) = (a \vee b) \vee c$. 5

   ii) Let *f* be a monomorphism from the lattice *L* into the lattice *M*. Show that *L* is isomorphic to a sublattice of *M*. 5

   b) i) In any lattice *L*, prove that $((x \wedge y) \vee (x \wedge z)) \wedge ((x \wedge y) \vee (y \wedge z)) = x \wedge y$ for all $x, y, z \in L$. 5

   ii) Show that the closed elements in a closure system of a complete lattice form a complete lattice. 5

   c) i) Simplify the Boolean polynomial $xy' + x(yz)' + z$.

   ii) Find the conjunctive normal form of $(x_1 + x_2 + x_3)(x_1x_2 + x_1'x_3)'$ 4+6

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