U.G. 2nd Semester Examination - 2022

MATHEMATICS

[HONOURS]

Course Code : MATH-H-CC-T-04

Full Marks : 60 Time : $2\frac{1}{2}$ Hours

The figures in the right-hand margin indicate marks.

The symbols and notations have their usual meanings.

1. Answer any **ten** questions: 2×10=20

a) Consider $\frac{dy}{dt} = f(y)$, where f(y) is defined by

$$f(y) = \begin{cases} 1, & y \ge 0 \\ -1, & y < 0, \end{cases}$$

and y(0)=0. Prove or disprove that there is no solution to this problem.

b) Determine the Wronskian W for the set of functions $\{x^3, |x^3|\}$ on [–1, 1].

c) Solve $\frac{dy}{dx} = \frac{2 + ye^{xy}}{2y - xe^{xy}}$.

d) Convert $\frac{dy}{dx} = \frac{xy^2 - y}{x}$ into an exact differential equation.

e) Solve $\frac{dy}{dx} - \frac{3}{4}y = x^4 y^{\frac{1}{3}}$.

f) What constant interest rate is required if an initial deposit placed into an account that accures interest compounded continuously is to double its value in six years? Solve by constructing differential equation.

g) Two solutions of $y'' - 2y' + y = 0$ are e^{-x} and $5e^{-x}$. Verify whether $y = c_1 e^{-x} + c_2 5e^{-x}$ is the general solution of the equation for the constants c_1 and c_2.

h) Show that all the orbits of the system

$$\dot{x}_1 = -x_2^3 \text{ and } \dot{x}_2 = x_1^3$$

are closed and surround the origin.

i) Solve the equation

$$\ddot{y}\dot{y} = t(1+t),\ y(0)=1,\ \dot{y}(0) = 2.$$

j) Consider the equation $\dot{y} = 3y^{\frac{2}{3}}$, y(0)=0. Show that the equation has infinitely many solutions.

k) Determine whether x=0 is an ordinary point or a singular point of the equation

$$2x^2\left(\frac{d^2y}{dx^2}\right)+7x(x+1)\left(\frac{dy}{dx}\right)-3y=0.$$

l) If S is defined by the rectangle $|x|\le a$, $|y|\le b$, then verify whether the function $f(x, y)=x\sin y+y\cos x$, satisfy Lipschitz condition or not.

m) Find the orthogonal trajectories of the family of cardioides $r=a(1-\cos\theta)$ for different values of a.

n) Form the partial differential equation by eliminating the constants a, b from

$$\theta=a\alpha+b\gamma+ab,$$

where θ is a function of α and γ.

o) Find the general integral of the partial differential equation

$$y^2p-xyq=x(z-2y).$$

2. Answer any **four** questions: 5×4=20

a) Solve $\ddot{y}-6\dot{y}+25y=2\sin\frac{t}{2}-\cos\frac{t}{2}$, by the method of undetermined coefficients.

b) Solve $x^3\frac{d^3y}{dx^3}+2x^2\frac{d^2y}{dx^2}+2y=10\left(x+\frac{1}{x}\right)$.

c) Obtain the power series solution of the equation

$$y''+(x-1)y'+y=0$$

in powers of (x–2).

d) Solve $y'''+y'=\sec x$ by the method of variation of parameter.

e) Find the characteristics of the equation pq=z and hence, determine the integral surface which passes through the parabola x=0, y^2=z.

f) Find the complete integral of the following PDE by Charpit's method:

$$(p^2+q^2)y=qz.$$

3. Answer any **two** questions: 10×2=20

a) i) Solve:

$$(x+a)^2\frac{d^2y}{dx^2}-4(x+a)\frac{dy}{dx}+6y=x.$$

ii) Show that

$$\sin x\frac{d^2y}{dx^2}-\cos x\frac{dy}{dx}+2y\sin x=0$$

is exact and solve it completely. 5+5

b) i) Solve the system of equations:

$$\frac{dx}{dt} + 4x + 3y = t$$

$$\frac{dy}{dt} + 2x + 5y = e^{t}$$

ii) Show that the equations

$$x^2 \frac{d^2y}{dx^2} + 2\left(x^3 - x\right)\frac{dy}{dx} + \left(1 - 2x^2\right)y = 0$$

and $$x^2 \frac{d^2z}{dx^2} + 2\left(x^3 + x\right)\frac{dz}{dx} - \left(1 - 2x^2\right)y = 0$$

have the same invariant and find the relation that transforms one into the other. 5+5

c) i) Solve $u_t - u_{xx} = 0$, $0 < x < L$, $0 < t < \infty$ satisfying

$$u(0,\ t) = u(L,\ t) = 0$$

$$u(x,\ 0) = f(x),\ 0 \le x \le L.$$

ii) Find the general solution of $y'' - xy' + 2y = 0$ near x=0. 5+5

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