KANDI RAJ COLLEGE – DEPARTMENT OF MATHEMATICS – INTERNAL EXAMINATION 2021 – 2ND SEMESTER – HONOURS

FULL MARKS [MATHEMATICS HONOURS]: CC – T – 03 = 10 ; CC – T – 04 = 10

FULL MARKS [HONOURS STUDENTS OF OTHER SUBJECTS]: GE – T – 02 = 10

FOR MATHEMATICS HONOURS STUDENTS

ANSWER CC-T-03 & CC-T-04 IN SEPARATE ANSWER SCRIPTS

	CC – T – 03	10
	Answer any TWO (2) questions	2 × 5
1.(i)	Show that the sequence $\{u_n\}$ where $u_n = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}$ is divergent.	3
(ii)	Show that the sequence $\{u_n\}$ where $u_n = \frac{sin\frac{n\pi}{2}}{n}$ is convergent and converges to 0.	2
2.	Test the series $1 + \frac{2x}{2!} + \frac{3^2x^2}{3!} + \frac{4^3x^3}{4!} + \cdots$	5
3.	State and prove the Archimedean property of real number.	5

CC – T – 04 — 10

Answer ALL questions

1. Find the exact solution of the initial value problem $\frac{dy}{dx} = y^2, y(0) = 1$. Starting with $y_0(x) = 1$, apply Picard's method to calculate $y_1(x), y_2(x), y_3(x)$ and compare these results with the exact solution. — 3

2. Show that $f(x,y) = x^2|y|$ satisfies a Lipschitz condition on the rectangle $|x| \le 1$ and $|y| \le 1$, but that $\frac{\partial f}{\partial y}$ fails to exist at many points of this rectangle. — 3

3. Solve the following differential equation — 2

$$\frac{dx}{z} = \frac{dy}{0} = \frac{dz}{-x}$$

4. Determine the nature and stability properties of the critical point $(0,0)$ of the following linear autonomous systems: — 2

$$\begin{cases} \frac{dx}{dt} = 2x \\ \frac{dy}{dt} = 3y \end{cases}$$

FOR MATHEMATICS HONOURS STUDENTS, THE QUESTION ENDS HERE

FOR STUDENTS, OTHER THAN MATHEMATICS HONOURS

GE – T – 02 — 10

Answer ALL questions

1. Solve — 3

$$(1 + y^2)dx = (\tan^{-1} y - x)dy$$

2. Solve — 2

$$(y - px)(p - 1) = p, \qquad \text{where}, p = \frac{dy}{dx}$$

3. Solve — 3

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = e^x$$

4. Solve — 2

$$\frac{dx}{y+z} = \frac{dy}{z+x} = \frac{dz}{x+y}$$

FOR OTHER HONOURS STUDENTS, THE QUESTION ENDS HERE